Scattered-Field Modeling in Linear Dispersive Media Using ADE-TLM Algorithm

H. El Faylali, M. Iben Yaich and M. Khaladi

Abstract: In this paper, we propose a scattered-field formulation for modeling dispersive media using the Transmission Line Matrix method with the symmetrical condensed node (SCN -TLM) and novel voltage sources. This model is based on the Auxiliary Differential Equation (ADE) technique. The proposed model, named ADE-TLM, is applied to Lorentz media for a scattered field TLM model

Keywords: **Auxiliary Differential Equation (ADE) technique, Finite-Difference Time Domain (FDTD), Transmission Line Matrix (TLM), dispersion, Lorentz.**

I. INTRODUCTION

By incorporating judiciously the polarization vector and Maxwell's equations, The FDTD method based on the auxiliary differential equation (ADE) technique has effectively modeled the interaction of femtosecsond pulses in silica fibers [1], [2]. The Z-transform technique combined with the FDTD method, gave consistent results for the simulation of solitons in nonlinear dispersive media [3], [4]. In order to simulate the wave's propagation in dispersive media with complex permittivity, methods based on the discrete convolution of the dispersion relation are presented in [5].The nonlinear Schrödinger equation was applied to model the propagation of short optical pulses in Kerr media [6]. The Alternating-Direction Implicit Finite-Difference Time-Domain (ADI-FDTD) has been introduced as an unconditionally stable implicit method to solve the problems of EM radiation in nonlinear dispersive media [7].

Front of the variety of numerical techniques employed and the rigor of the proposed approaches and models developed by the FDTD method to simulate electromagnetic (EM) wave propagation in nonlinear media and predict their interactions, the TLM method provides further opportunities for proposals and development of new modeling tools and investigation despite of the numerous studies performed in this field [8]-[12].

In this paper, we increase the applicability of the TLM numerical method to model the linear dispersion properties. A novel algorithm based on the TLM method with condensed symmetrical node (SCN) and new voltage sources combined with the auxiliary differential equation ADE, is used to model the Lorentz dispersion in linear optical media. In our proposed model, we reformulate the auxiliary differential equation (ADE) technique in the context of the TLM method.

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This approach allows the simulation of the propagation of optical pulses in linear Lorentz media, by solving Maxwell's and polarization equations describing the effects of the media on the EM wave's propagation. This approach is based on the time discretization applied to the components of the EM field and polarization vector by using centered differencing.

II. FORMULATION

The Maxwell-Ampere equation for an EM wave propagating along the z-direction (i.e., E_x component) in a dispersive linear medium can be written as follows:

$$\frac{\partial \bar{E}_x(t)}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon_\infty}\left[-\frac{\partial \bar{P}_x(t)}{\partial t} + \left(\nabla \Lambda \bar{H}\right)_x\right] \quad (1)$$

Where ε_0 is the dielectric constant of free space, ε_∞ is the relative dielectric constant in the limit of infinite frequency and $P_x(t) = P_{Lorentz}(t)$ is the linear polarization term. E_x and P_x are respectively the components, along the x-axis, of the electric field and polarization vectors.

Using centered differencing scheme between time steps $t^{n+1} = (n+1)\Delta l$ and $t^n = n\Delta l$, we can write the discretized equation for the electric and polarization fields in regular space $(\Delta x = \Delta y = \Delta z = \Delta l)$ as follows:

$$E_x^{n+1} = E_x^n - \frac{1}{\varepsilon_0 \varepsilon_\infty}(P_x^{n+1} - P_x^n) + \frac{\Delta t}{\varepsilon_0 \varepsilon_\infty}\left(\nabla \Lambda H^{n+\frac{1}{2}}\right)_x \quad (2)$$

To develop a TLM model for the numerical treatment of these problems we need to transform the EM field parameters into circuit parameters that are related to transmission lines. Substituting in "(2)", the component E_x by its electric equivalent $\frac{V_x}{\Delta l}$, we obtain the expression of the discretized electric voltage component V_x^{n+1} along the x-axis at time step t^{n+1} :

$$V_x^{n+1} = V_x^n - \frac{\Delta l}{\varepsilon_0 \varepsilon_\infty}(P_x^{n+1} - P_x^n) + \frac{\Delta t \Delta l}{\varepsilon_0 \varepsilon_\infty}\left(\nabla \Lambda H^{n+\frac{1}{2}}\right)_x \quad (3)$$

III. SECOND-ORDER LINEAR DISPERSION

The Lorentz polarization as a function of electric voltage is governed by the following auxiliary differential equation:

$$\frac{\partial^2 P_{Lorentz}(t)}{\partial t^2} + 2\delta_{Lorentz}\frac{\partial P_{Lorentz}(t)}{\partial t} + \omega_{Lorentz}^2 P_{Lorentz}(t) = \frac{\varepsilon_0(\varepsilon_{Lorentz} - \varepsilon_\infty)}{\Delta l}\omega_{Lorentz}^2 V(t) \quad (4)$$

Where $\omega_{Lorentz}$ is the Lorentz characteristic resonant frequency, $\delta_{Lorentz}$ the damping factor and $\varepsilon_{Lorentz}$ the static permittivity caused by the Lorentz dispersion.

By using centered differencing, the polarization of the Lorentz dispersion $P_{Lorentz}^{n+1}$ at time step t^{n+1} obeys to the following difference equation:

$$P_{Lorentz}^{n+1} = a_{Lorentz}P_{Lorentz}^{n} + b_{Lorentz}P_{Lorentz}^{n-1} + c_{Lorentz}\frac{V_x^n}{\Delta l} \quad (5)$$

Where

$$a_{Lorentz} = \frac{2-\omega_{Lorentz}^2\Delta t^2}{1+\delta_{Lorentz}\Delta t}$$

$$b_{Lorentz} = -\frac{1-\delta_{Lorentz}\Delta t}{1+\delta_{Lorentz}\Delta t}$$

$$c_{Lorentz} = \frac{\varepsilon_0(\varepsilon_{Lorentz}-\varepsilon_\infty)\omega_{Lorentz}^2\Delta t^2}{1+\delta_{Lorentz}\Delta t}$$

IV. ADE-TLM ALGORITHM

By substituting the terms of the polarization given by "(5)", we can write the discretized linear relation describing electrical voltage at time step t^{n+1} as:

$$V_x^{n+1} = V_x^n - \frac{\Delta l}{\varepsilon_0\varepsilon_\infty}(P_{Lorentz}^{n+1} - P_{Lorentz}^{n}) + \frac{\Delta t\Delta l}{\varepsilon_0\varepsilon_\infty}\left(\nabla\Lambda H^{n+\frac{1}{2}}\right)_x \quad (6)$$

Writing the magnetic field components of "(6)" in terms of incident and reflected pulses [13]-[14], then applying the charge conservation principle for the symmetrical condensed node SCN with a capacitive stub of normalized admittance $Y_{ox}(t)$ we obtain the following expression for the discretized electric voltage V_x^{n+1} at time step t^{n+1}:

$$V_x^{n+1} = \frac{2}{4+Y_{ox}}{}_{n+1}\left(V_1^i + V_2^i + V_9^i + V_{12}^i + Y_{ox}V_{13}^i + 0.5V_{sx}\right) \quad (7)$$

V_{sx} and Y_{ox} are respectively the voltage source, and the normalized admittance terms in order to take into account, in the TLM mesh, linear properties. They are given by:

$$V_{sx} = -{}_nV_{sx} + 4\left[\varepsilon_\infty V_x^n - \frac{\Delta l}{\varepsilon_0}\left(P_{Lorentz}^{n+1} - P_{Lorentz}^{n}\right)\right] \quad (8)$$

$${}_{n+1}Y_{ox} = 4\left(\varepsilon_\infty - 1\right) \quad (9)$$

The ADE-TLM model with voltage sources is based on recursive calculation of normalized admittance made in "(9)" and the voltage sources expressed in "(8)". The obtained values are then inserted in "(6)". The solution of the last equation is then used in the calculation of reflected pulses and in the connection process along the TLM mesh nodes.

V. NUMERICAL RESULTS

In order to show the efficiency of the new model ADE-TLM with voltage sources, we present spatial variations of electrical field propagation in a medium having a Lorentz linear dispersion characterized by the following [15]:

$$\varepsilon_s = 5.25,\ \varepsilon_\infty = 2.25,\ \omega_{Lorentz} = 0.4\times10^{15}\,\text{rad/s and}\ \delta_{Lorentz} = 0.1\omega_{Lorentz}.$$

The considered TLM mesh dimensions are (1, 1, 5000) Δl with space step $\Delta l = 8$ nm. The air-nonlinear dispersive medium interface is located at $z = 8\Delta l$ and excited by Gaussian unit amplitude pulse. Recall that when an EM wave propagates in a medium, the field induces a time varying dipole moment in the individual atoms that comprise the medium. The oscillating atoms lose energy through radiative and nonradiative mechanisms. The implied polarization is expressed through changes in the reflected field.

Figure 1 shows the spatial evolution to the right of the short pulse propagation, after 8000, 16000 and 32000 iterations for a linear dispersive medium. Our results are in accordance with those presented in [11] and [15].

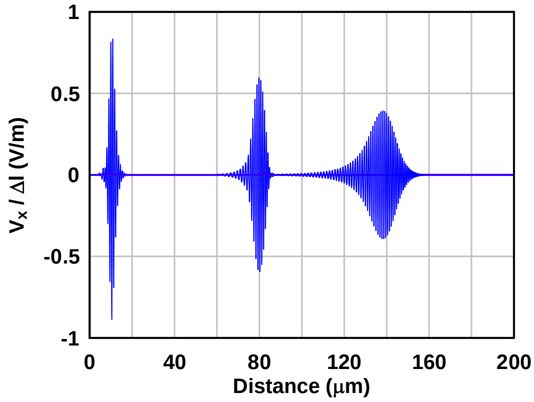

Fig. 1. TLM results of the short pulse propagation after 8000, 16000 and 32000 time steps respectively in Lorentz medium.

Figure 2 illustrates the time evolution of the reflected electric field in Lorentz medium. These results reproduce those of [16].

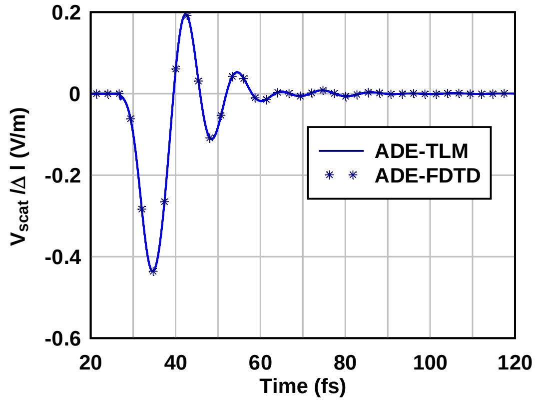

Fig. 2. Comparison of the reflected electric field in Lorentz media as computed by ADE-TLM and ADE-FDTD methods**.** located at $z = 18\Delta l$.

Figure 3 compares the ADE-TLM calculated and exact theoretical results [17] for the magnitude of the reflection coefficient for Lorentz media.

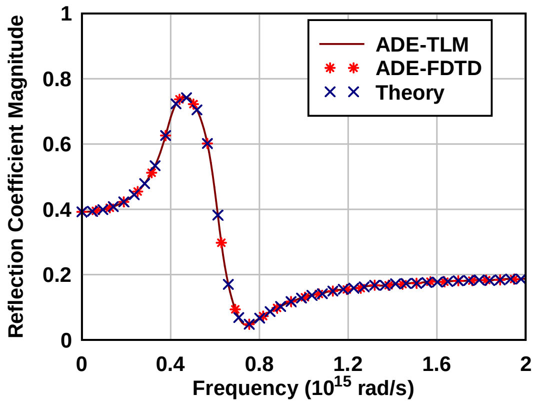

Fig. 3. Comparison of the reflection coefficient magnitude for the Lorentz medium as predicted by exact analytical results, FDTD and ADE-TLM methods.

VI. Conclusion

We have efficiently analyzed Maxwell's equations for modeling propagation of reflected optical pulses in linear dispersive media using the ADE-TLM model. In this model we introduced a voltage sources modeling linear properties and the variable admittance concept.

An excellent agreement was obtained between the model ADE-TLM and the theoretical results or existing methods.

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