Review On: Design of Low Complexity LDPC Decoding Algorithm in LDPC Coded BICM-OFDM Systems

Tarannum Shaikh and Prof. Devichand P. Rathod

Abstract : Digital Video Broadcasting-Terrestrial (DVB-T) is the most widely deployed digital terrestrial television system worldwide with services on air in over thirty countries. In order to increase its spectral efficiency and to enable new services the DVB consortium has developed a new standard named DVB-T2. Latest and next-generation wireless broadcasting standards, such as DVB-T2 or DVB-NGH, are considering distributed multi-antenna transmission in order to increase bandwidth efficiency and signal quality. Full-rate full-diversity (FRFD) space-time codes (STC) such as the Golden code and LDPC code are studied for that purpose. Bit-interleaved coded modulation (BICM) schemes based on low-density parity check (LDPC) codes have been used to enhance the detection complexity. A novel low-complexity soft detection algorithm for the reception of Golden codes in LDPC based orthogonal frequency-division multiplexing (OFDM) systems is presented in this paper. Computational complexity investigation as well as simulation results indicate that this algorithm has significant performance and complexity advantages over existing optimal detection in the various DVB-T2 broadcasting scenario.

Keywords –Bit Interleaved Coded Modulation (BICM), Low-density parity check (LDPC) codes, MAP detection, MIMO systems, Orthogonal frequency division multiplexing (OFDM).

I. INTRODUCTION

Since the invention of Information Theory by Shannon in 1948, coding theorists have been trying to come up with coding schemes that will achieve capacity dictated by Shannon's Theorem. The most successful two coding schemes among many are the LDPCs and Turbo codes. This article presents LDPC codes and in particular their usage by the second generation terrestrial digital video broadcasting (DVB-T2) [1], DVB-T2 specification makes use of LDPC (Low density parity-check) codes in combination with BCH (Bose-Chaudhuri- Hocquengham) to protect against high noise levels and interference. The second generation of the terrestrial digital video broadcasting standard (DVB-T2) [1] has adopted a space-frequency block code (SFBC) based on the well-known Alamouti technique. In order to increase the capacity and reach the full multiple-input multiple-output (MIMO) diversity-multiplexing frontier, the proposals for the future generations of terrestrial, portable and mobile digital video broadcasting standards, such as DVB-NGH, focus on the combination of both diversity and spatial multiplexing [2, 3] through full-rate full diversity (FRFD) codes such as the Golden code.

The main advantage of LDPC codes is that they provide a performance which approaches the channel capacity for many different scenarios, as well as the linear algorithms that can be used for decoding. Actually, the efficiency improvement provided by DVB-T2 in comparison with DVB-T is mainly based on these new coding and interleaving schemes. LDPC codes are commonly decoded by a soft-input soft- output (SISO) algorithm. When iterative decoders, such as turbo or low-density parity check (LDPC) codes, are included in the reception chain, soft information on the conditional probabilities for all possible transmitted symbols is required in the form of log-likelihood ratios (LLR). Several algorithms that serve this purpose can be found in the literature for spatial multiplexing MIMO systems, mostly based on list sphere detection (LSD) [4].
This paper presents the design of a low-complexity soft detection algorithm for Golden codes in bit-interleaved coded modulation (BICM) OFDM systems based on LDPC coding. We design the tree search algorithm in order to find the best balance between complexity and performance, by analyzing different tree configuration. Results based on a the DVB-T2 transmission scheme show how the proposed receiver design and algorithms can make the processing of Golden codes feasible for close-to-optimal soft decoding.

II. BLOCK DIAGRAM

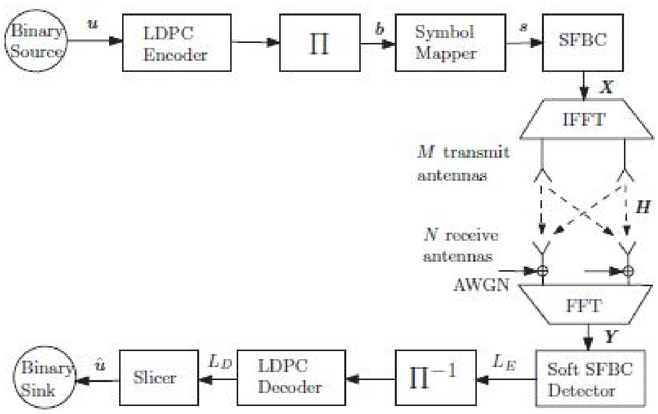

Fig. 1. Simplified diagram of a LDPC-based MIMO transmission and reception scheme based on DVB-T2.

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Fig. 1 shows the basic structure of LDPC-coded BICM-OFDM scheme with two transmit (M = 2) and two receive (N = 2) antennas. As can be seen, the bit stream is coded, interleaved and mapped onto a complex constellation which is spread across the transmit antennas and consecutive subcarriers through the Golden code. If subcarriers are grouped according to the chosen space-frequency mapping, a frequency-domain received symbol block $\boldsymbol{Y}$ of dimensions $N \times T$ can be represented mathematically as

$$\boldsymbol{Y} = \boldsymbol{HX} + \boldsymbol{Z}, \qquad (1)$$

where $\boldsymbol{H}$ denotes the $N \times M$ complex channel matrix, $\boldsymbol{X}$ is any $M \times T$ codeword matrix and $\boldsymbol{Z}$ represents the $N \times T$ zero-mean additive white Gaussian noise (AWGN) matrix whose complex coefficients fulfill $CN(0, 2\sigma^{2)}$ being σ^2 the noise variance per real component. Note that T = 2is the frequency depth of the considered codewords $\boldsymbol{X}$. The Golden code is the 2 × 2 FRFD scheme which achieves the maximal coding gain [6]. A data symbol vector $\boldsymbol{s}$ = (s1, s2, s3, s4) is transformed into the transmitted codeword as follows:

$$\boldsymbol{X} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha\left(s_1 + \theta s_3\right) & \alpha\left(s_2 + \theta s_4\right) \\ i\bar{\alpha}\left(s_2 + \bar{\theta} s_4\right) & \bar{\alpha}\left(s_1 + \bar{\theta} s_3\right) \end{bmatrix}, \qquad (2)$$

with $\theta = \frac{1+\sqrt{5}}{2}$ (the golden number), $\bar{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1 + i - i\theta$ and $\bar{\alpha} = 1 + i - i\bar{\theta}$. However, the symbols are not dispersed with equal energy in all spatial and temporal directions in such a way that the symbol power within each of the pairs, (s1, s3) and (s2, s4), are always unbalanced.

The main drawback of the Golden code lies on the decoding complexity, which is exponential in the length of the symbol vector s, i.e. $O(P^4)$, becoming prohibitive for large constellation sizes P.

III. SOFT DETECTION

The aim of a soft-output detector is to calculate or approximate the *a posteriori* probability (APP) for each of the coded bits cj in a given signaling interval, where $j \in \{1, \ldots .\omega Nt\}$ is the bit index. This probability is conveniently represented by the so-called a posteriori log-likelihood ratio (LLR):

$$L_D(b_k|\boldsymbol{Y}) = \ln \frac{\Pr\left[b_k = +1|\boldsymbol{Y}\right]}{\Pr\left[b_k = -1|\boldsymbol{Y}\right]} = L_A(b_k) + L_E(b_k|\boldsymbol{Y}), \quad (3)$$

The sign of $L_D(b_k|\mathbf{y})$ is the maximum a posteriori (MAP) estimate for b_k, and the magnitude represents the reliability of the estimate. Larger magnitudes correspond to higher reliability, and smaller magnitudes indicate low reliability. In particular, the extreme case of L = 0 indicates that b_k is equally likely to be +1 and −1.where Pr [b_k = +1] and Pr [b_k = −1] are the *a priori* probabilities that bit b_k is 1 or −1, respectively, and where

$$LA(bk) = \ln \frac{\Pr\ [bk = +1\ ]}{\Pr\ [bk = -1]}$$

is the a priori LLR for the k-th bit. The second term LE in (3) represents the extrinsic contribution to the a posteriori LLR [4]. Using the law of total probability, it can be written as

$$L_E(b_k|\boldsymbol{Y}) = \ln \frac{\sum\limits_{\boldsymbol{b} \in \mathbb{B}_{k,+1}} p(\boldsymbol{Y}|\boldsymbol{b}) \exp\left(\sum\limits_{j \in \mathbb{J}_{k,\boldsymbol{b}}} L_A(b_j)\right)}{\sum\limits_{\boldsymbol{b} \in \mathbb{B}_{k,-1}} p(\boldsymbol{Y}|\boldsymbol{b}) \exp\left(\sum\limits_{j \in \mathbb{J}_{k,\boldsymbol{b}}} L_A(b_j)\right)}, \qquad (4)$$

where $LD(bk)$, $LA(bk)$ and $LE(bk|\boldsymbol{Y})$ denote the *a posteriori*, *a priori* and *extrinsic* information, respectively. The extrinsic information conditioned to the received vector $\boldsymbol{Y}$ is written in (4).

where $p(\boldsymbol{Y}|\boldsymbol{b})$ represents the likelihood function. Defining $K_b = MT \log 2\, P$, B_k, +1 represent the set of $2^{Kb}-1$ bit vectors $\boldsymbol{b}$ having $b_k = +1$.

The most important part of the calculation of LD in (3) is the likelihood function $p(\boldsymbol{Y}|\boldsymbol{b})$. Considering our system in (1), we can rewrite it as an equivalent 4 × 4 MIMO channel where there is no channel interference between the sets of transmit antennas {1, 2}, {3, 4} and the sets of receive antennas {3, 4}, {1, 2}, respectively.

Thus, the equivalent channel can be expressed as

$$\breve{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H}^1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{H}^2 \end{bmatrix} = \begin{bmatrix} h_{11}^1 & h_{12}^1 & 0 & 0 \\ h_{21}^1 & h_{22}^1 & 0 & 0 \\ 0 & 0 & h_{11}^2 & h_{12}^2 \\ 0 & 0 & h_{21}^2 & h_{22}^2 \end{bmatrix}, \qquad (5)$$

where h_{ij}^k is the complex channel coefficient from transmit antenna j to receive antenna i at the k-th carrier. Note that we have distinguished between $\boldsymbol{H}^1$ and $\boldsymbol{H}^2$ since they are equal if and only if the channel does not vary in adjacent carriers.

By taking the elements column-wise from matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, the column vectors $\boldsymbol{x}$ = [x11, x21, x12, x22]T and $\boldsymbol{y}$ = [y11, y21, y12, y22]T can be defined, respectively.

On the other hand, if we define a generator matrix $\boldsymbol{G}$ for the Golden code as

$$\boldsymbol{G} = \begin{bmatrix} 1 + i\bar{\theta} & 0 & \theta - i & 0 \\ 0 & -\theta + i & 0 & 1 - i\bar{\theta} \\ 0 & 1 + i\bar{\theta} & 0 & \theta - i \\ 1 + i\theta & 0 & \bar{\theta} - i & 0 \end{bmatrix}, \qquad (6)$$

the codeword $\boldsymbol{X}$ can be expressed as $\boldsymbol{x} = \boldsymbol{Gs}$, where $\boldsymbol{s}$ corresponds to the symbol column vector [s1, s2, s3, s4]T . Using the new notation, the likelihood function $p(\boldsymbol{Y}|\boldsymbol{b})$ can be rewritten as

$$p(\boldsymbol{Y}|s = map(\boldsymbol{b})) = \frac{\exp\left(-\frac{\left\|\boldsymbol{y} - \breve{\boldsymbol{H}}\boldsymbol{G}\boldsymbol{s}\right\|^2}{2\sigma^2}\right)}{\left(2\pi\sigma^2\right)^{NT}}, \qquad (7)$$

where $\boldsymbol{s} = map(\boldsymbol{b})$ is the mapping of the vector $\boldsymbol{b}$ into the symbols of column vector $\boldsymbol{s}$. Only the term inside the exponent in (7) is relevant for the calculation of LE, and the constant factor outside the exponent can be omitted.

The main difficulty in the calculation of (4) arises from the computation of the ML metrics since a calculation of P4 metrics is necessary for the considered 2 × 2 code. This becomes unfeasible for high modulation orders unless the calculation of (4) can be reduced.

This detection approach results in a good system performance and its complexity depends on the method to obtain the candidate list *L*. The LSD is the most common approach, but its complexity order will be upper-bounded by *O*(*P*4) in the same way as the SD [4]. In the next section, we propose a fixed-complexity detection algorithm and its design for FRFD codes such as (2).

IV. FIXED-COMPLEXITY DETECTION

The proposed fixed-complexity tree-search-style algorithm is based on the functionality of list fixed-complexity sphere decoder (LFSD) presented in [7] for spatial multiplexing schemes with soft symbol information requirements. The main feature of the LFSD is that, instead of constraining the search to those nodes whose accumulated Euclidean distances are within a certain radius from the received signal, the search is performed in an unconstrained fashion. The tree search is defined instead by a tree configuration vector ***n*** = [*n*1, . . . , *nMT*], which determines the number of child nodes (*ni*) to be considered at each level. At the end of the process, a list of *Ncand* candidate symbol vectors is retrieved from the last-level nodes that have been reached. It is worth noting that the set G composed of the *Ncand* selected symbol vectors may not correspond to the vectors of the *L* set with the smallest metrics given by the LSD, but provides sufficiently small metrics and diversity of bit values to obtain accurate soft information. A representation of an LFSD tree search is depicted in Figure 2 for a QPSK modulation and a tree configuration vector of ***n*** = [1, 1, 2,4].

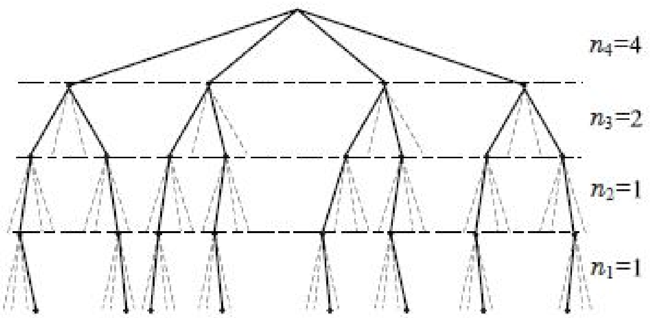

Fig. 2. Fixed-complexity tree search of a QPSK-modulated signal using a tree configuration vector of ***n*** = [1, 1, 2, 4].

The basic idea behind the Fixed Sphere Decoder is to perform a search over only a fixed number of possible transmitted signals, generated by a small subset of all possible signals located around the received signal vector. This ensures that the detector complexity is fixed over time, a major advantage for hardware implementation. In order for such a search to operate efficiently, a key point is to order the antennas in such a way that most of the points considered relate to transmit antennas with the poorest signal to noise (SNR) conditions.

4.1 Ordering algorithm

A simple ordering algorithm is used that exploits the golden code's structure to ensure that the overall algorithm performs well. Our detector has computational complexity O(*q*2) when the list length ℓ = *q*2. However, for ℓ << *q*2, our algorithm achieves comparable performance at much lower complexity.

The first step in the proposed algorithm is to determine which pair of information symbols should be detected first, and to furthermore determine the order in which the remaining pair of symbols is to be detected. The pseudo code for an efficient ordering algorithm, is shown below

Ordering Algorithm.

Input: **H** **Output**: **P**

if $\|\mathbf{h}_1\| > \|\mathbf{h}_2\|$ **then**
  $k_1 = 1$
  $C_2 = [2, 3, 4], C_3 = [4, 4, 2], C_4 = [3, 2, 3]$
else
  $k_1 = 2$
  $C_2 = [1, 3, 4], C_3 = [3, 1, 3], C_4 = [4, 4, 1]$
end
$\delta_{min} = \infty$
for *i from* 1 *to* 3 **do**
  $k_2 = C_2[i], k_3 = C_3[i], k_4 = C_4[i]$
  $\delta = |\mathbf{h}_{k_1}^{*}\mathbf{h}_{k_2}|/(\|\mathbf{h}_{k_1}\|\|\mathbf{h}_{k_2}\|)$
  if $\delta < \delta_{min}$ **then**
    $\delta_{min} = \delta$
    $\mathbf{P} = [\mathbf{e}_{k_1}, \mathbf{e}_{k_2}, \mathbf{e}_{k_3}, \mathbf{e}_{k_4}]$
  end
end

The performance of the LFSD soft-detector in uncoded scenarios is strongly dependent on the ordering algorithm of the channel matrix and the choice of the tree configuration vector [8]. However, in the specific case of space-frequency-coded systems the effect of the ordering algorithm on the overall performance relies on the symbol power distribution in spatial and frequency directions. The structure of the Golden code generates a difference in the norms of the equivalent subchannels of each symbol in a pair, which allows for the implementation of an ordering procedure in order to improve the overall system's performance. The proposed ordering approach yields close-to-optimum performance when combined with the suggested tree configuration vector. Moreover, the matrix ordering process only requires the computation of *MT* vector norms as opposed to other ordering algorithms such as FSD [7] which need to perform *MT* − 1 matrix inversion operations.

V. SIMULATION RESULTS

The performance of the overall system has been assessed by means of the bit error rate (BER) after the LDPC decoder. The DVB-T2 parameters used in the simulations are: 64800 bits of length of the LDPC block, code rate R =2/3, 16-QAM modulation, OFDM symbol of 2048 carriers (2K) and 1/4 of guard interval. The simulations have been carried out over a Rayleigh channel (Typical Urban of six path, TU6), commonly used as the simulation environment for terrestrial digital television systems. Perfect CSI and non-iterative detection has been considered at the receiver.

5.1. Performance comparison over DVB-T2 BICM system

This section presents the performance assessment and the complexity analysis of the new list fixed-complexity soft detector of the LDPC code over a SFBC DVB-T2 broadcasting scenario. Fig. 3 shows the BER curves versus SNR for different configurations of the proposed algorithm. for the LFSD configuration k = 1, p = P and *Ncand* = 25. However, this grows up to 1.3 dB with a less complex configuration, i.e. k = 2, p = 8. On the other hand, one should note that the LSD performance difference between *Ncand* = 25 and 50 is negligible resulting advantageous in the complexity degree of the algorithm.

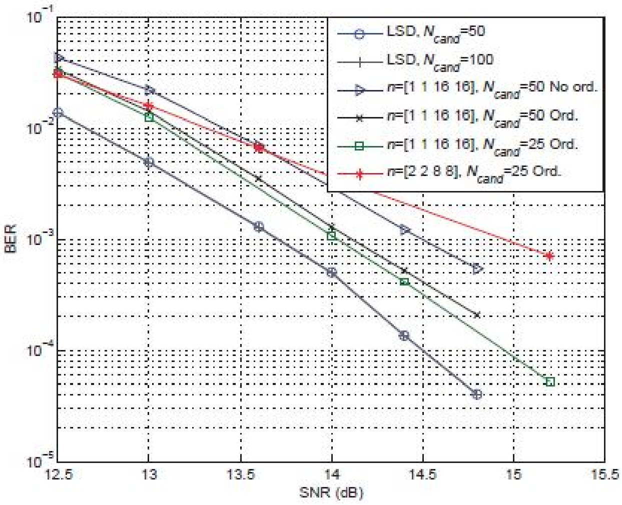

Fig. 3. BER performance comparison between LSD and LFSD detection of LDPC codes in the 2 × 2 DVB-T2 system with 16-QAM modulation over TU6 channel.

The BER performances of the proposed algorithm, the LFSD of [8] and the LSD solution are depicted in Fig. 4 for *Ncand* = 50. One can observe that the proposed fixed complexity detection algorithm achieves a similar performance result as the LSD with a substantial reduction in the detection complexity. Moreover, the new ordering design and the proposed tree search configuration vector ***n*** outperform the LFSD solution of [8] in about 0.7 and 1 dB for Golden and SS codes, respectively. If we observe the behavior of the proposed fixed-complexity algorithm for the SS code, we can see that it obtains the same BER performance as the algorithm proposed in [7], which has a complexity of $O(P^3)$, with complexity P^2. For the Golden code, if the fixed-complexity tree of P^2 branches is considered, the performance is 0.4 dB worse than the LSD with complexity $O(P^4)$. However, if the complexity is increased to $4P^2$, the performance difference is negligible.

In order to analyze the complexity of the detection algorithms, the cumulative distribution functions of the overall visited nodes have been depicted in Fig. 4. We see that the reduction of *Ncand* decreases the complexity of the LSD decoder compared to the LFSD. For *Ncand* = 50, 75 % of the LSD solutions are obtained visiting lower number of nodes than the LFSD algorithm.

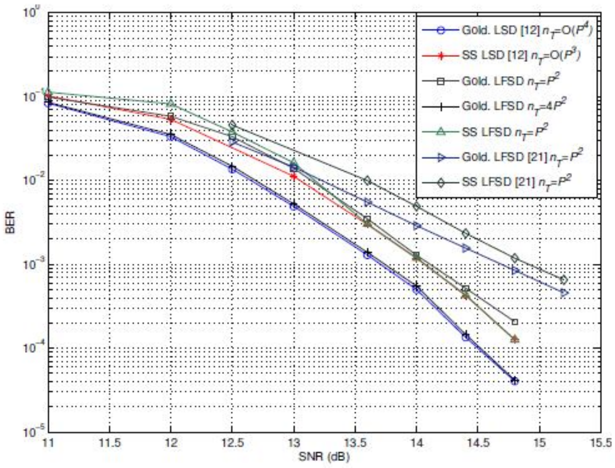

Fig . 4. BER performance comparison of FRFD SFBC codes detection with LSD [9], LFSD of [11] and the proposed ordered LFSD for DVB-T2 transmission.

If *Ncand* is reduced up to 25, this value rises to 95 %. Despite these differences, the sequential nature of the LSD tree search and its variable complexity results in a problem for real hardware implementations. Nevertheless, the design of LFSD makes it possible a parallel architecture of the algorithm that can be fully pipelined and maintains fixed the search complexity.

VI. Conclusion

In this paper, a fixed-complexity detector for Golden codes and the analysis of its implementation on future digital TV broadcasting systems based on LDPC-coded BICM-OFDM is presented. The main drawback of the LSD detection is its variable complexity that is strongly dependent on the noise and channel conditions, which leads to a complexity order upper-bounded by $O(P^4)$. A list fixed-complexity detector with a novel ordering algorithm is proposed in this paper with the aim of approaching the performance of the LSD using fixed complexity.BER simulation results show the close-to-optimal performance of the proposed low-complexity detector in a typical LDPC-based DVB-T2 broadcasting scenario. The proposed detection algorithm can enable the realistic implementation and the inclusion of LDPC codes in the forthcoming digital video broadcasting standards or in any similar BICM-OFDM system. BER simulation results show the close-to-optimal performance of the proposed low-complexity detector for both Golden and SS SFBC codes in a typical LDPC-based DVB-T2 broadcasting scenario. The performance is clearly improved when the proposed channel and candidate ordering algorithm is applied with Golden codes, though its effects are negligible for the SS code. In any case, the proposed detection algorithm can enable the realistic implementation and the inclusion of any FRFD SFBC code in any

BICM-OFDM system such as the forthcoming digital video broadcasting standards or in any similar BICM-OFDM system.

ACKNOWLEDGMENT

I would like to express my deepest gratitude to my guide Prof. D.P.Rathod for his support and guidance. It was his enthusiastic encouragement that led me to work with him, then his appreciation led me to further successes in my academic career. I also would like to thank him for his suggestion and support to write this Paper.

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