Liquid level control of coupled tank system using Fractional PID controller

Arun Kumar, Munish Vashishth and Lalit Rai

Abstract—**In process industries PID controllers are used more. In PID controller we have three parameters for tuning and tuning is done by using Ziegler-Nicholas method. In this paper we used fractional PID controller for coupled tank system and by using a tuning method for getting required response. First used zieglar-nicholas and Astrom-Hagglund method for tuning, then derived two non-linear equations and then solve these equations to find fractional order of the derivative and integral terms. In this paper we used a optimization strategy to obtained required step response of given process.**

Keywords—**FOPID, PID, SISO, fractional order controllers.**

I. Introduction

PID Controller is used in most common in process industry applications. Main reason of using PID controller is its simple structure and easily implementation. Fractional control algorithm for control of dynamic system has been introduced and performance of CRONE over PID Controller has been demonstrated by oustaloup.CRONE is used for MIMO system. We use here NINTEGER toolbox.Podlubny has proposed a PID controller as P$I^{\lambda}D^{\mu}$controller which is known as fractional order controller. Where lambda is non-integer order of integrator and mu is noninteger order of differentiator. P$I^{\lambda}D^{\mu}$ controller has better response than PID controller.FOPID controllers receives considerable attention in last years from academic and industrial point of view. Fractional order controllers provides flexibility in controller design with respect to standard PIDcontrollers, because they have five parameters instead of three in PIDcontroller. In this paper we have two tanks connected in series and control the liquid level coupled tank using fractional PIDcontroller. We used tuning method[6] for fractional PID controller is a new method used for PID controller one is Ziegler-Nicholous method and second one is Astrom-Hugglund method , and fractional derivative and fractional intregral order is find by using fsolve function in optimization toolbox in MATLAB. This paper uses the new tuning method[7] for coupled tank system.

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II. Fractional Calculus

In elementary calculus we familiar to differential operator D=$\frac{d}{dx}$.fractional calulus deals with $D^n f(x) = \frac{d^n f(x)}{dx^n}$ called differ-integral operator and n is a positive integer. Fractional order controller based on concept of fractional calculus. The continuous integro-differential operator is defined as [1]

$$ {}_aD_t^q = \begin{cases} \frac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \int_a^t d\tau^{-q} & q < 0 \end{cases} \quad (1)$$

Where a and t bound of operation and q is the fractional order which can be a complex number.

In literature there are several definitions are available.Grunwald-Letinkov definition is

$$ {}_aD_t^q f(t) = \frac{d^q f(t)}{d(t-a)^q} = \lim_{n\to\infty} \left[\frac{t-a}{N}\right]^{-q} \sum_{j=0}^{N-1} (-1)^j \binom{q}{j} f\left(t - j\left[\frac{t-a}{N}\right]\right) \quad (2)$$

The Riemann-Liouville definition is the simplest and easiest definition to use. This definition is given by

$$ {}_aD_t^q f(t) = \frac{d^q f(t)}{d(t-a)^q} = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t (t-\tau)^{n-q-1} f(\tau) d\tau \quad (3)$$

Where n is the first integer which is not less than q i.e. n-1<q<n and Γ is the Gamma function,

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad (4)$$

III-Fractional order controller (P$I^{\lambda}D^{\mu}$)

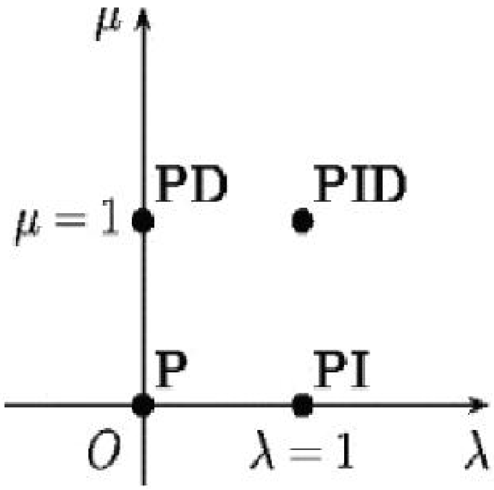
The most common form of a fractional order PID controller is the P$I^{\lambda}D^{\mu}$ controller, involving an integrator of order λ and a differentiator of order μ where μ and λ can be any real numbers. The transfer function of such a controller has the form[5]

$$G_c(s) = \frac{U(s)}{E(s)} = k_P + k_I \frac{1}{s^{\lambda}} + k_D s^{\mu}, \quad (\lambda, \mu > 0) \quad (5)$$

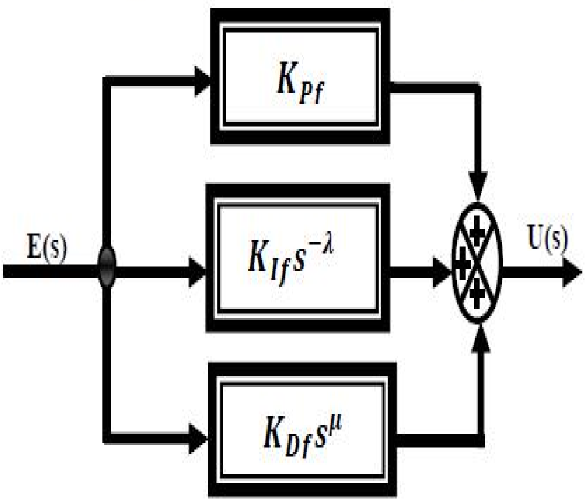
Where G_c(s) is the transfer function of the controller, E(s) is an error and U(s) is controller's output. The integrator term is$1/s^{\lambda}$, that is to say, on a semi-logarithmic plane, there is a line having slope -20λ dB/decade. The control signal u(t) can then be expressed in the time domain as

$$u(t) = k_P e(t) + k_I D^{-\lambda} e(t) + k_D D^{\mu} e(t) \quad (6)$$

Figure is a block-diagram configuration of FOPID. Clearly selecting λ=1 and μ=1, a classical PID controller can be recovered. The selections of λ =1, μ=0 and λ =0, μ=1 respectively corresponds conventional PI & PD controllers. All these classical types of PID controllers are the special cases of the fractional $PI^{\lambda}D^{\mu}$ controller given by



FigI-Expension of PID from point to plane



FigII-structure of FOPID controller

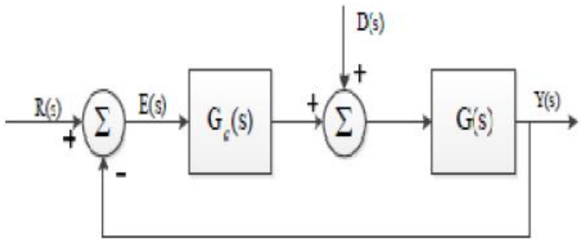


Fig III – Unity feedback control scheme

The considered control scheme; G(s) is the process, $G_c(s)$ is the FOPID controller, R(s) is the reference input, E(s) is the error, D(s) is the disturbance and Y(s) is the output.

For design FOPID controller we first find the order of differentiation and order of integrator. These are positive real numbers. Apart from conventional PID controller FOPID Controller is generalization of PID Controller.

FOPID controllers enhance the system control performance.one of the advantage of $PI^{\lambda}D^{\mu}$ is better control of dynamical system. Due to two more extra degree of freedom FOPID controller is less sensitive to change of parameters of controlled system.

IV-COUPLED TANK SYSTEM

In this system we have two tanks and both tanks are connected in series. These are same as interacting quantity[2].here we control the liquid level of tank2 by use the input flow rate(Fin).

Coupled tank system diagram is shown in figure. We considered as SISO process .input flow rate in tank1 is manipulated variable and liquid level of tank2 is the control variable.

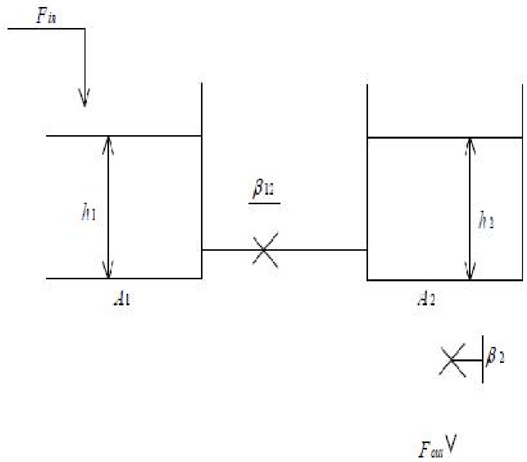


Fig IV- coupled tank SISO process

Two nonlinear equations are obtained in terms of input and output variable using Bernoulli's law[4]

$$\frac{dh_1(t)}{dt} = -\frac{\beta_{12}a_{12}}{A_1}\sqrt{2g(h_1(t) - (h_2(t))} + \frac{k}{A_1}u(t) \quad (7)$$

$$\frac{dh_2(t)}{dt} = -\frac{\beta_2 a_2}{A_2}\sqrt{2g(h_2(t))} + \frac{\beta_{12}a_{12}}{A_2}\sqrt{2g(h_1(t) - h_2(t))} \quad (8)$$

Where

A_1&A_2-cross sectional area of tank1 and tank2

a_1-cross sectional area of output pipe in tank2

a_{12}-cross sectional area of interaction pipe b/w tank1 and tank2

h_1&h_2-water level of tank1 and tank2

F_{in}-inflow(lph)

F_{out}-outflow(lph)

β_{12}-valve ratio of jointed pipe b/w tank1 and tank2

β_2-valve ratio at outlet of tank2

K-gain of pump
g-gravity
u-input voltage
after linearization of eq.(7) & eq.(8),we get

$$\frac{dH_1(t)}{dt} = \frac{1}{T_{12}}\left(-H_1(t) + \left(H_2(t)\right)\right) + \frac{k}{A_1}U(t) \quad (9)$$

$$\frac{dH_2(t)}{dt} = \frac{1}{T_2}\left(-H_2(t)\right) + \frac{1}{T_{12}}\left(H_1(t) - H_2(t)\right) \quad (10)$$

Where

$$T_{12} = \frac{A_1}{\beta_{12}a_1}\sqrt{\frac{2(h_1 - h_2)}{g}}, s$$

$$T_2 = \frac{A_2}{\beta_2 a_2}\sqrt{\frac{2h_2}{g}}, s$$

T_{12}--- time constant between tank1 and tank2

T_2---- time constant of tank2

$$\frac{H_2(s)}{U(s)} = G(s) = \frac{K}{T_{12}T_2 s^2 + (T_{12} + 2T_2)S + 1} \quad (11)$$

Where

$$K = \frac{kT_2}{A_2}, cm/V$$

Physical parameters for coupled tank system is defined as[3]

Table I-Parameters for couple tank system

Parameters	Values
Tank Cross Section Area	A=154
Pipe area cross section	a=0.5
Tank Water Levels	29.55 cm, 24.66 cm
Tank valve ratio	0.6820043, 1.5315195
Input voltage	u=2.5 V
Gravity	g=981 cm^2/s
Pump gain	K=30.0024

put above values in eq(11),we get transfer function

$$G(s) = \frac{19.7277}{2033.30981s^2 + 222.60117s + 1}$$

V- TUNING METHOD OF FOPID CONTROLLER

First compute the parameters of FOPID controller[6],then following steps are followed

- Find value of phase margin.
- Obtain k_p and k_i from the Ziegler- Nichols tuing rules
- Obtain non lineareqs.[7]
- Specify the initial value for k_d using Astrom-Hagglund method
- Find the numerical solutions for ƛ and μ from non lineareqs. [7], considering new value of K_D.
- If the step response of the system is not satisfactory enough, an optimization can be done by using the optimization model to obtain better values for the controller parameter

VI-SIMULATION RESULT AND DISCUSSION

Fig shows Simulink model for coupled tank system for FOPID controller

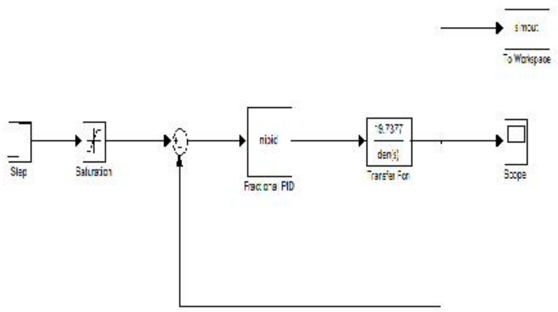


Fig V-SIMULINK MODEL OF COUPLED TANK

A FOPID controller is designs for integer order model of coupled tank system using NINTEGER TOOLBOX in MATLAB.

After using the tuning method[6] .parameters of FOPID controller are

$\boldsymbol{K_P}$**=24.2318**

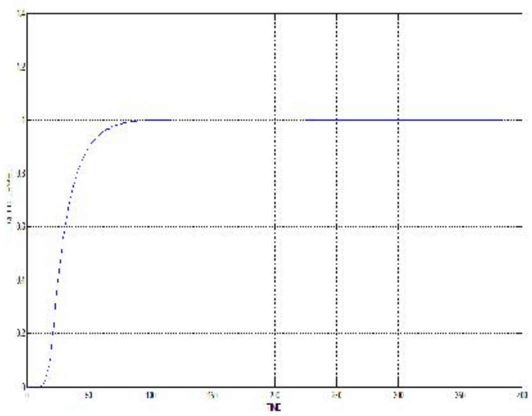
$\boldsymbol{K_I}$**=256.13**

$\boldsymbol{K_D}$**=1.4302**

ƛ = 0.9578

μ =0.8664

put these values in nipid block in Simulink we get the step response



Peak response=143 sec

Rise time=99.7 sec

Settling time=109 sec

Steady state value=1

If same system is simulated using PID Controller then system settled after more time than FOPID controller.

VI-CONCLUSION

In present work performance of FOPID controller is shown here.first the Simulink model of coupled tank system is constructed with help of Simulink .then the performance of FOPID controller is simulated.no overshoot is presented in response of FOPID controller with coupled tank system.it gives better performance than PID controller due to two more tuning parameters present in FOPID controller comparison to PID controller.

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