SER Analysis of Iterative Channel Estimation Based on Least Square Method in OFDM

Ravi R. Patel and Prof. Vijay K. Patel

Abstract: Orthogonal frequency division multiplexing (OFDM) provides an effective and low complexity means of eliminating inter symbol interference for transmission over frequency selective fading channels. In OFDM systems, channel estimation is very crucial to demodulate the data coherently. For good channel estimation, spectral efficiency and lower computational complexity are two important points to be considered. In this paper, we explore Iterative channel estimation based on Least Square (LS) method in order to improve Symbol Error Rate (SER) by applying the feedback for OFDM system. We first investigate simple pilot based least square channel estimation technique and its performance. Then, in order to improve the performance we propose an iterative channel estimation and data detection technique by adding virtual pilots. Using iterative method we get improved Symbol error rate performance than conventional channel estimation method.

Keywords: **Channel Estimation, Iterative Channel Estimation, Least square method and Orthogonal Frequency Division Multiplexing.**

I. INTRODUCTION

Currently, *orthogonal frequency-division multiplexing* (OFDM) systems are subject to significant investigation. Since this technique has been adopted in the European digital audio broadcasting (DAB) system [1], OFDM signaling in fading channel environments has gained a broad interest [2]. For instance, its applicability to digital TV broadcasting is currently being investigated [3].

The use of *differential phase-shift keying* (DPSK) in OFDM systems avoids the tracking of a time varying channel [4]. However, this will limit the number of bits per symbol and results in a **3** dB loss in *signal-to-noise ratio* (SNR) [5]. If the receiver contains a channel estimator, multi amplitude signaling schemes can be used.

In [6] and [7], 16-QAM modulation in an OFDM system has been investigated. A decision-directed channel tracking method, which allows the use of multi-amplitude schemes in a slow Rayleigh-fading environment, is analyzed in [6].

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In the design of wireless OFDM systems, the channel is usually assumed to have a finite-length impulse response. A cyclic extension, longer than this impulse response, is put between consecutive blocks in order to avoid inter block interference and preserve orthogonality of the tones [8]. Generally, the OFDM system is designed so that the cyclic extension is a small percentage of the total symbol length.

In Section II, we describe the system model. Section III discusses the *least-squares* (LS) and minimum *mean-square error* (MMSE) channel estimators. The LS estimator has low complexity, but its performance is not as good as that of the MMSE estimator while the MMSE estimator has good performance but high complexity. In Section IV we present Iterative technique to the LS estimators that use the virtual pilots. In Section V we evaluate the estimators by simulating a BPSK signaling scheme. The performance is presented in terms of *symbol error rate* (SER).

II. SYSTEM MODEL

Figure 1 shows the OFDM base band mode where $\boldsymbol{x}$ is the transmitted signal, $\boldsymbol{y}$ is the received signal, $g(t)$ is the channel impulse response. Here, we use AWGN channel model and noise is the white Gaussian channel noise. A cyclic prefix (which is not shown in Figure 1) is used to preserve the orthogonality of OFDM consecutive blocks and to avoid the inter-symbol interference between the consecutive OFDM blocks.

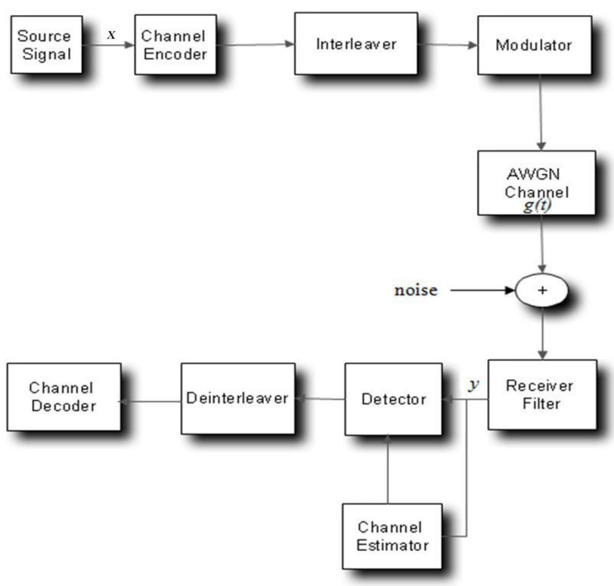
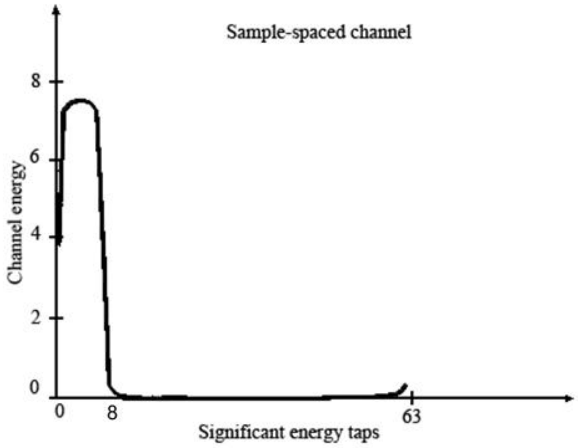


Fig.1: Block diagram of OFDM channel estimator and detector

We consider a channel impulse response which is consists of *0 to N numbers* and of *0 to N non-* zero pulses. The impulse response of the channel is

$$g(t)=\sum_{m=0}^{N} i_m \delta(\tau-\tau_m T_s) \quad (1)$$

where, i_m is the zero mean complex Gaussian random variable, ***g(t)*** is treated as the time limited pulse train, T_S is the sampling period, τ_m is the delay of N^{th} impulse, where the first delay $\tau_0 = 0$ and others impulse delay have been uniformly distributed over the length of the cyclic prefix.



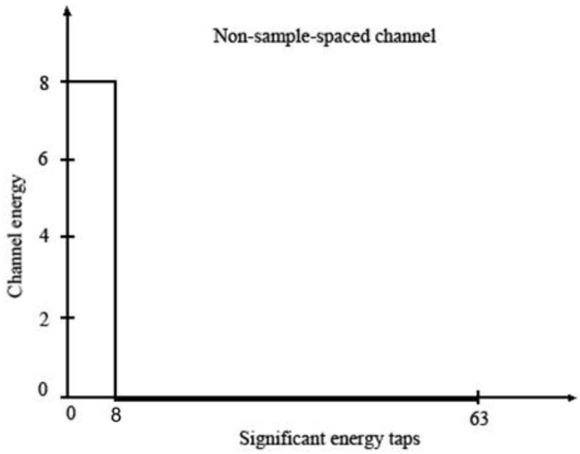


Fig 2**:** Sample spaced channel and non sampled spaced channel.

In the OFDM system, the channel impulse response time duration is less than the OFDM symbol time. The channel transfer function or channel attenuation ***h*** absorbs most of the channel energy within few samples. Figure 2 shows the energy absorption in time domain for two types of channel, the sample spaced channel and the non sample spaced channels. The sample spaced channel is a channel whose impulse response is finite and multiple of the system sampling rate, for that reason the DFT gives the optimal energy concentration [9]. For the non-sample-spaced channel, the IDFT of the channel attenuation ***h*** does not confine with cyclic prefix, because the channel attenuation ***h*** is the continuous Fourier transform of the channel ***g***. Although the IDFT of ***h*** does not confine with cyclic prefix but it preserves the orthogonality. The requirement for orthogonality is that the continuous time channel has length that is shorter than the cyclic prefix. By using the *N*-point discrete time Fourier transform (DFT) system can be modeled as

$$y = DFT_N[IDFT_N(x) * \frac{g}{\sqrt{N}} + \tilde{n}] \quad (2)$$

Here $x=[x_0,x_1,,,,,x_{N-1}]^T$, $y=[y_0,y_1,,,,,y_{N-1}]^T$, $\tilde{n}=[n_0,n_1,,,,,n_{N-1}]^T$ and * is called the cyclic convolution and $g=[g_0,g_1,,,,,g_{N-1}]^T$ is determined by sinc functions [1].

$$g_k=\frac{1}{\sqrt{N}}\sum_{m=0}^{N} i_m e^{-j\frac{\pi}{N}}(k+(n-1)\tau_m)\frac{\sin(\pi\tau_m)}{\sin(\frac{\pi}{N}(\tau_m-k))} \quad (3)$$

So, the amplitude i_m are the complex valued and $0 \leq \tau_m T_S \leq T_g$, k is an integer value range of 0 to N-1, if τ_m is an integer then all the energy from, i_m is mapped to taps g_k but when τ_m is not an integer then τ_m energy will leak to all taps of g_k. Usually most of the energy is located near the original pulse location. The overall system can be written as

$$y_k = x_k h_k + n_k \quad (4)$$

Here, k = 0, 1, 2…………*N*-1. h_k is the channel attenuation vector for 0 to N-1 channel and g_k is the channel energy for 0 to N-1 channel, where $h_k=[h_0,h_1,,,,,h_{N-1}]^T = DFT(g_k)$

For simplicity we may rewrite equation (14) as following

$$y = xFg + n \quad (5)$$

F is the DFT matrix.

III. CHANNEL ESTIMATION

The channel estimator provides the knowledge on the Channel Impulse Response (CIR) to detectors. The channel estimation is based on the known sequence of bits which is unique for a particular transmitter and which is repeated in every transmission burst. The channel estimator is able to estimate the CIR for each burst separately by the exploiting transmitted bits and the corresponding received bits.

A. MMSE estimator

The MMSE estimator major rule is to efficiently estimate the channel to minimize the MSE or SER of the channel. In equation (6), R_{gg} and R_{yy} denote as the auto-covariance matrix of ***g*** and ***y*** respectively, where ***g*** is the channel energy and ***y*** is the received signal. Moreover, the cross covariance of ***g*** and ***y*** is denoted by R_{gy} and the noise variance ***E*** $\{|N|^2\}$ is denoted by δ^2_n. The channel estimation by using MMSE estimator g_{MMSE} can be derived as follows:

$$g_{MMSE} = R_{gy}R_{yy}^{-1}y \quad (6)$$

Where,

$$R_{gy} = E\{gy^H\} = R_{gg}F^H x^H \quad (7)$$

$$R_{yy} = E\{yy^H\} = xFR_{gg}F^H x^H + \delta_n^2 I_n \quad (8)$$

The columns in ***F*** are orthogonal and ***I*** is the identity matrix.

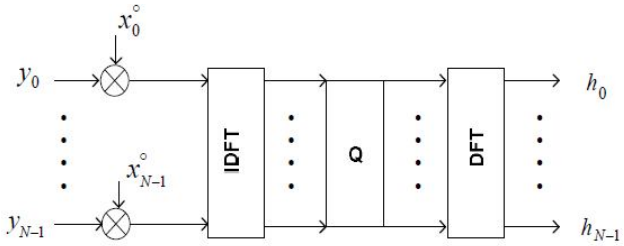


Fig 3: Block diagram of channel estimator.

From Figure 3, the channel impulse response h_{MMSE} is as follows:

$$h_{MMSE} = Fg_{MMSE} = FQ_{MMSE}F^H x^H y \quad (9)$$

Where,

$$Q_{MMSE} = R_{gg}[(F^H x^H xF)\delta_n^2 + R_{gg}]^{-1}(F^H x^H Fx)^{-1} \quad (10)$$

h_{MMSE} is the channel attenuation for MMSE estimator, g_{MMSE} is the channel energy, $\boldsymbol{y}$ is received signal, $\boldsymbol{x}$ is the transmitted signal and $\boldsymbol{F}$ is the DFT matrix [1].

B. LS Estimator

The LS estimator has lower computational complexity than MMSE. The LS estimator for the cyclic impulse response $\boldsymbol{g}$ minimizes $(y\text{-}xFg)$ $(y\text{-}xFg)^H$ and generates the channel attenuation as bellow

$$h_{LS} = FQ_{LS}F^H x^H y \quad (11)$$

Here,

$$Q_{LS} = (F^H x^H Fx)^{-1} \quad (12)$$

And $(y\text{-}xFg)^H$ is the conjugate transpose operations. So, the lest square h_{LS} can be written as

$$h_{LS} = x^{-1}y \quad (13)$$

Where, the least square h_{LS} is the channel attenuation for LS. Equations (9) and (13) are the general expressions for MMSE and LS estimators respectively. Both estimators have some own drawbacks. However the MMSE estimator performance is better but computational complexity is high, contrary the LS estimator has high mean-square error but its computational complexity is very low. For reducing computational complexity and improve performance, we have two channel estimation approaches.

1. Mean square error

The mean square error or MSE of an estimator is one of many ways to quantify the difference between the theoretical values of an estimator and the true value of the quantity being estimated. MSE measures the average of the square of the error. The error is the amount by which the estimator differs from the quantity to be estimated. We define the mean square error as

Mean square error = mean [{abs (H) –abs ($h_{estimator}$)}^{2}] (14)

Where, $\boldsymbol{H}$ is theoretical transfer function and $h_{estimator}$ is the calculated transfer function for each estimator.

2. Symbol Error Rate

Symbol rate is the number of symbol changes made to the transmission medium per second using a digitally modulated signal. Symbol error rate for 16-QAM system is [10]

$$P_{s,16-QAM} = \frac{3}{2} erfc(\sqrt{\frac{E_s}{10N_0}} \quad (15)$$

Where, erfc denoted complementary error function, E_s denoted signal energy and N_0 denoted bit rate.

IV. ITERATIVE CHANNEL ESTIMATION

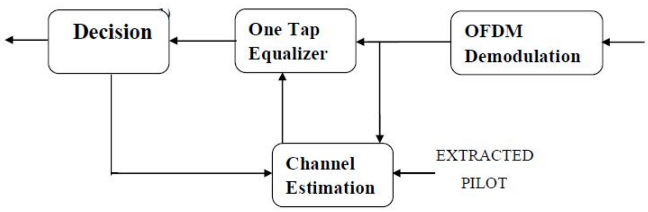


Fig 4: OFDM receiver for iterative channel estimation

The channel estimation accuracy can be improved by adding virtual pilots using an iterative channel estimation and data detection. The hard decision symbols can be used as virtual pilots. Thus, there will be iteration between the decision and the channel estimation block at the receiver, which is a kind of decision feedback equalization technique, as seen in Figure-4. Initially channel is estimated using LS method which gives initial estimation now this estimated data is detected and again fed back for iterative estimation that gives the improved performance than the conventional LS method. This improved performance is closer to the MMSE method.

V. SIMULATION RESULTS

In the simulations we consider a system operating with a bandwidth of 500 kHz**,** divided into 64 tones with a total symbol period of 138 *μs***,** of which 10 *μs* is a cyclic prefix.
Sampling is performed with a 500 kHz rate. A symbol thus consists of 69 samples, five of which are contained in the cyclic prefix (i.e. $L = 5$). Figure 5 shows the comparison between the LS and MMSE estimator based on SER versus SNR. In the SNR range from 2 dB to 20 dB, the MMSE estimator SER is lower than the LS estimator. SERs of LS and MMSE are almost the same from 25 dB SNR range. Figure 6 shows the comparison between LS and iterative LS estimator based on SER versus SNR. From the result we can see that the SER of iterative LS is lower than conventional LS method. So SER is improved using iterative method.

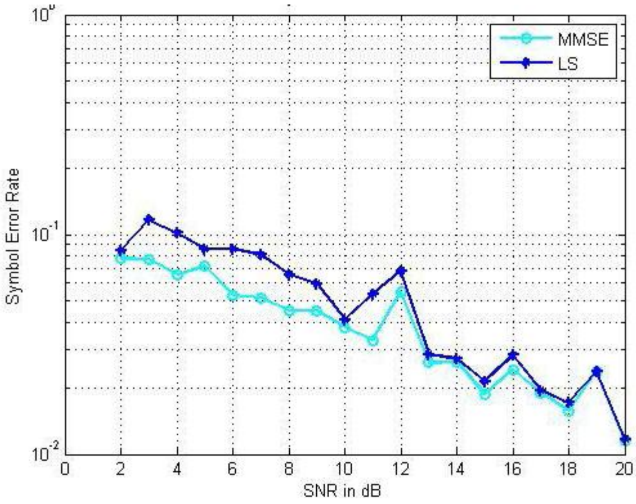


Fig 5: Performance analysis for MMSE and LS based on SER versus SNR

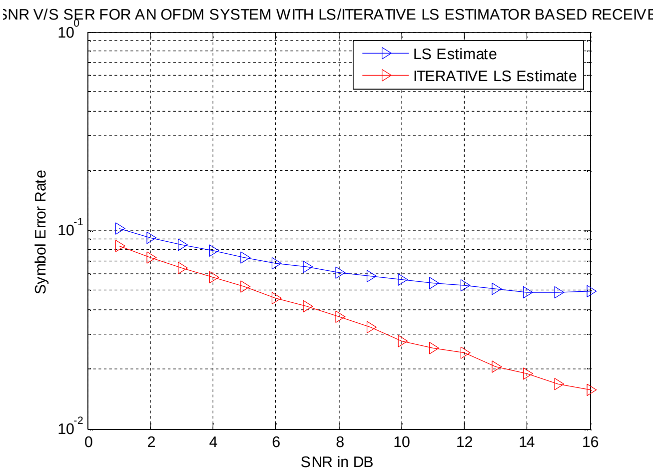


Fig 6: Performance analysis of LS and iterative LS based on SER versus SNR

VI. CONCLUSION

In this paper, performance analysis of iterative channel estimation based on LS method for OFDM system is given. Firstly, we show the general structure of the LS and the MMSE estimator performances. Based on the performance analysis the MMSE estimator is recognized as better than LS estimator, but the MMSE estimator suffers from high computational complexity. After that we introduce iterative LS method to improve the SER. From the result we can conclude that iterative LS method has improved performance which is closer to the MMSE estimation method.

REFERENCES

[1] Jan-Jaap van de Beek, Ove Edfors, Magnus Sandell Sarah Kate Wilson and Per Ola Borjesson, "On Channel Estimation in OFDM systems" In proceedings Of VTC'95 Vol. 2 pg.815-819

[2] Leonard J. Cimini, Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency-division multiplexing", IEEE Duns. Comm., Vol. 33, no. 7, pp 665-675, July 1985

[3] M. Alard and R. Lassalle, "Principles of modulation and channel coding for digital broadcasting for mobile receivers", EBU Review, no. 224, pp 3-25, August 1987

[4] B. Marti et al., "European activities on digital television broadcasting-from company to cooperative projects", *EBU* Technical Review, no. 256, pp. 20-29, 1993

[5] John Proakis, Digital Communications, McGraw-Hill, 1.989.

[6] Sarah Kate Wilson, R. Ellen Khayata and John M. Cioffi, "16-$AM modulation with orthogonal frequency-division multiplexing in a Rayleigh-fading environment", In Proc. VTC-1994, pp. 1660-1664, Stockholm, Sweden, June 1994.

[7] Peter Hoeher, "TCM on frequency-selective land-mobile fading channels", In Proc. of the 5th Tirrenia International Workshop on Digital Communications, Tirrenia, Italy, September 1991.

[8] John A.C. Bingham, "Multicarrier modulation for data transmission: an idea whose time has come", IEEE Communications Magazine, 28(5):5-14, May 1990.

[9] Ove Edfors, Magnus Sandell, Jan-Jaap van de Beek, Sarah Kate Wilson and Per Ola Borjesson, "Analysis of DFT based channel estimator for OFDM", IEEE Transactions on Wireless Personal Communications, vol. 12, pp. 55-70, 2000.

[10] ''Symbol error rate for 16-QAM''.Available: http://en.wikipedia.org/wiki/Symbol-error-rate-for-16-qam

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