

# Algorithm for Removal of DC Offset in Current and Voltage Signals

Shilpi Nayak, Shraddha Kaushik and Ishawari Prasad Sahu

**Abstract:** Protecting transmission lines frequently involves adopting distance relays. Protecting relays must filter their inputs to reject unwanted quantities and retain signal quantities of interest. This paper presents a new algorithm suitable for calculating impedances from digitized voltages and currents sampled at a relay location. Each input is assumed to be composed of a decaying d. c. component and components of the fundamental and harmonic frequencies. Parameters of a digital filter determined by using the least error squares approach are then used to compute the real and imaginary components of the voltage and current phasors. Impedances as seen from a relay location are then calculated.

**Keywords—** Digital filtering, distance relay, decaying DC component, discrete fourier transform.

## I. INTRODUCTION

Transmission lines form a major part of a power system. Different types of relays are used to protect these lines. The most commonly used relays are from the family of distance relays. One of these relays, the impedance relay is used on its own or in conjunction with communication facilities for communicating with the relays at the remote terminal of the line. These relays basically evaluate the impedance looking into the transmission line from the voltages and currents at the relay location. The impedance is assumed to be proportional to the distance from the relay to the fault and, this determines if the fault is in the relay's protective zone.

The conventional impedance relays are of either electromechanical or solid state (electronic) type. Some relays which use a digital processor for computing the impedance and making decisions have been developed in the last few years. The algorithms used to calculate the apparent impedance used in these relays can be categorized into four groups. The first group is developed assuming that the waveforms presented to the relay are pure sinusoids. The second group of algorithms use Fourier analysis and the third group use digital filters to extract the fundamental frequency information from the inputs. The last group of algorithms numerically solve a differential equation which describes the behaviour of the transmission line.

Mann and Morrison proposed an algorithm which uses the fact that the amplitude of a sinusoid can be determined from its value and its rate of change at any instant and the rate of change of a function can be calculated by using difference equations. Gilcrest, Rockefeller and Udren used the first and second derivatives to calculate the peak values of the sinusoids. Miki and Makino expressed the peak squared values of the voltage and current in terms of two sampled values, each of the voltage and current. The calculated peak values were then used to compute the real and reactive components (R and X) of the apparent impedance. Gilbert and Shovlin calculated R and X directly from the sampled values except that three sets of samples were used instead of two sets used in reference 3. The second group of algorithms use some form of Fourier analysis to extract the fundamental frequency information. Ramamoorthy suggested an algorithm which samples the signals over one cycle of the fundamental frequency and determines the real and imaginary parts of the fundamental frequency component. Using sampling intervals of ninety degrees at the fundamental frequency in the algorithm provides the Carr and Jackson algorithm. Phadke, Hlibka and Ibrahim used this approach and one half cycle data window in their algorithm. The third type of algorithms use digital filters to extract information concerning the fundamental frequency components; Hope and Umamaheswaran used finite impulse response filters. The last group of algorithms have been proposed and used by McInnes and Morrison, Ranjbar and Corylo and Breingan. These algorithms numerically solve the differential equation representing the transmission line by a series R-L model.

This paper presents a new algorithm which is based on the least error squares curve fitting technique. The algorithm assumes that the input is composed of a fundamental frequency component, a decaying d. c. and harmonics of specified order. The decay rate of the d. c. component is not assumed in advance because it is affected both by the resistance of the arc at the fault and the effective resistance of the system. Mathematical background necessary for developing this algorithm is first presented. Effects of sampling frequency, data window, time reference and changing the model of the decaying d. c. component are then examined. The selected parameters are then used in the impedance calculating algorithm. Impedances calculated from the data of a single phase to ground fault remote from the relay and a close-in three phase fault on 138 kV lines over approximately three cycles after the inception of the faults are reported. The fault data which was recorded at the Regina South switching station of the Saskatchewan Power Corporation was used for these tests.

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Shilpi Nayak and Ishawari Prasad Sahu are working as Assistant Professors in Electrical and Electronics Engineering Department at Chhatrapati Shivaji Institute of Technology, Durg, Chhattisgarh and Shraddha Kaushik is current working an Assistant Professor in Electrical and Electronics Engineering Department at Rungta college of Engineering and Technology, Bhilai, (INDIA). Emails: [shilpi.nayak88@gmail.com](mailto:shilpi.nayak88@gmail.com), [shraddha.kaushik01@gmail.com](mailto:shraddha.kaushik01@gmail.com), [ishawarisahu@csitdurg.in](mailto:ishawarisahu@csitdurg.in)

II. MATHEMATICAL ALGORITHM

In all previously designed digital relays, voltage and current outputs of the transducers are preprocessed and converted to millivolts level. The function of the preprocessors is two fold; one is to suppress surges travelling from the power system to the relay and the other is to block high frequencies from reaching the relays. Preprocessed signals of the millivolt level are converted to numerical values by analog to digital converters. The digital information is then provided to a processor which analyzes the information and makes appropriate decisions. The mathematics of the proposed least error squares filter is presented in this section. The outputs of this filter are the real and imaginary components of the fundamental frequency phasor. These components are then used to calculate the impedance as seen from the relay location.

Calculating the Real and Imaginary Components of Voltages and Currents-

The output of a CCVT or voltage transformer during a fault is a waveform composed of a decaying d. c. component and many harmonic components. At time  $t=t_1$ , this waveform can be mathematically expressed as:

$$f(x) = A e^{-t/\tau} + \sum_{n=1}^M (A_n \sin(n \omega t + \phi_n)) \quad (1)$$

$A$  = magnitude of the decaying dc offset;

$\tau$  = time constant of the decaying dc offset;

$A_n$  = amplitude of the nth harmonics;  $\omega = 2\pi f$  ;

The time constant  $\tau$  depends on the X/R ratio of the system; but is also affected by the arc resistance which varies from fault to fault. In practice, even harmonics are not present in the fault voltages and currents. Also, higher order harmonics are blocked from reaching the relay by the signal conditioning equipment which usually includes analog filters. The cut-off frequency of these filters is determined by the overall design considerations for the relay.

So, equation (1) becomes;

$$f(x) = A e^{-t/\tau} + A_1 \sin(\omega t + \phi_1) + A_3 \sin(3 \omega t + \phi_3) \quad (2)$$

The exponential term  $e^{-t/\tau}$  of equation (2) can be expanded using Taylor's series, such that

$$e^{-t/\tau} = 1 - \frac{t}{\tau} + \frac{t^2}{2! \tau^2} - \frac{t^3}{3! \tau^3} + \dots \quad (3)$$

Using the first three terms of this series and assuming that

(i) the signal conditioning equipment has effectively blocked the fifth and higher order harmonics and (ii) no even harmonics are present in the input.

By considering only the first terms of this expression, equation (2) can be expressed as

$$f(x) = A - A \frac{t}{\tau} + A \frac{t^2}{2! \tau^2} + A_1 \sin(\omega t + \phi_1) + A_3 \sin(3 \omega t + \phi_3) \quad (4)$$

Equation (5) is obtained from expanding sine terms.

$$f(x) = A - A \frac{t}{\tau} + A \frac{t^2}{2 \tau^2} + A_1 \cos \phi_1 \sin(\omega t) + A_1 \sin \phi_1 \cos(\omega t) + A_3 \cos \phi_3 \sin(3 \omega t) + A_3 \sin \phi_3 \cos(3 \omega t) \quad (5)$$

This equation may be written in the more convenient form:

$$S_1 = f(x) = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 + a_{17}x_7 \quad (6)$$

Where  $S_1$  is sample measured at time  $t_1$ . The coefficient in equation (6) is related only on the time at which the samples are taken, and they form:

$$a_{11}=1; a_{14}=\sin(3 \omega t_1); a_{12}=\sin(\omega t_1); a_{15}=\cos(3 \omega t_1); a_{13}=\cos(\omega t_1); a_{16}=t_1; a_{17}=t_1^2;$$

The  $x$  values are functions of the unknown, and are given by:

$$x_1 = A; x_2 = A_1 \cos \phi_1; x_3 = A_1 \sin \phi_1; x_4 = A_3 \cos \phi_3; x_5 = A_3 \sin \phi_3; x_6 = -A/\tau; x_7 = A/\tau^2;$$

The samples measured at time  $t = t_2$  can likewise be expressed as:

$$S_2 = f(x) = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 + a_{27}x_7 \quad (7)$$

As mentioned previously, the  $a$ -coefficients are functions of time. Therefore if  $t_1$  is taken as a time references and the current is sampled at preselected times, then the values of the sampled  $t$  coefficient of equations (7) and (8) can be specified. So equations can be written in matrix form:

$$[A] [X] = [S] \quad (8)$$

From equation (8) we can find the value of matrix  $[X]$

$$[X] = [A]^{-1} [S] \quad (9)$$

$$[A]^{-1} = [A^T A]^{-1} [A^T] \quad (10)$$

Then we can find fundamental phasor from  $[X]$

$$A_1 = \sqrt{x_2^2 + x_3^2} \quad (11)$$

$$A = x_1 \quad (12)$$

### III. RESULT AND DISCUSSION

Fundamental Component	100.0000
DC Component	100.2782
Error DC Percentage	0.2782

From the above results we can say that least square technique is very effective method for computing fundamental and harmonics component and DC offset. It requires 0.205093 sec to execute full program in MATLAB.

This algorithm requires that digitized values of voltages and currents be multiplied by the elements of the second and third rows of an  $[A]^T$ . Two approaches can be used for this purpose. One could be to use a hardware multiplier. The strength of the proposed algorithm lies in two areas; the freedom in choosing the equation of condition and some flexibility in selecting the size of the data window. The freedom of choosing the equation of condition allows prespecified harmonics to be included in the equation. This enables the harmonic components to be determined for applications such as, transformer differential protection.

This algorithm explicitly takes account of the decaying d.c. component by including it in the equation of condition.

### IV. CONCLUSION

This paper describes the least error squares approach for developing a digital filter which explicitly takes account of the decaying d. c. components in the system voltages and currents. The concept of pseudo inverse which has been used in developing the algorithm is also presented. It is shown that the proposed approach can effectively calculate the impedance from the fault data obtained from a power system.

The technique presented in this paper is general enough to be extended and applied in situations, such as, transformer differential faults. If the harmonics observed in transformer in-rush currents are included in the equation of conditions, the resulting algorithm can provide information concerning the harmonic components for use in transformer differential relays.

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**Shilpi Navak** received her Bachelor degree in Electrical and Electronics Engineering from Chhatrapati Shivaji Institute Of Technology, Durg, Chhattisgarh (INDIA) in 2010. She is pursuing her Master of Engineering in Power Electronics (Electrical) from Rungta College of Engineering & Technology, Bhilai, Chhattisgarh (INDIA). She is currently working as an Assistant Professor in Electrical and Electronics Engineering Department at Chhatrapati Shivaji Institute Of Technology, Durg, Chhattisgarh, (INDIA). Her research interests includes Power Generation, Transmission, Distribution, and Power Electronics.

**Shraddha Kaushik** received her Bachelor degree in Electrical and Electronics Engineering from Government Engineering College, Raipur (INDIA) in 2009 and M.Tech in Power Systems and Automation from GITAM Institute of Technology, GITAM University Visakhapatnam, (INDIA) in 2012. She is currently working as an Assistant Professor in Electrical and Electronics Engineering Department at Rungta college of Engineering and Technology, Bhilai, (INDIA). Her research interests includes Distribution Generation, Smart Grids and Machine Designing.

**Ishawari Prasad Sahu** received his bachelor degree in Electrical and Electronics Engineering from Chhatrapati Shivaji Institute Of Technology, Durg, Chhattisgarh (INDIA) in 2010 and M.Tech in Integrated Power Systems from Visvesvaraya National Institute of Technology, Nagpur (INDIA) in 2012. He is currently working as an Assistant Professor in Electrical and Electronics Engineering Department at Chhatrapati Shivaji Institute Of Technology, Durg, Chhattisgarh (INDIA). His research interests include Power Transmission, Distribution, protection and FACTS.