A Method to Solve and Study the Time Delayed Vibration Characteristics of Smart Material Actuator Applying Lambert W-Function

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Abstract: **Actuator, sensor and devices that are made of smart materials exhibit time delayed response during actuation under stimulus or in sensing process. In this paper, a new method has been presented for solution of the time delayed vibration characteristics of active material actuator applying the special Lambert W-function. For this, a low cost silver-electrode ionic polymer metal composite (IPMC) actuator is studied both experimentally and theoretically. An analytical model has been developed based on input-output relationship that describes the time delayed vibration response of the actuator. Multi-modes excitation is assumed and Hamilton's principle is applied for deriving the delay differential equation (DDE) of the actuator. Lambert-W function is then applied and a closed form solution of the transcendental characteristic equation of delay differential equation (DDE) is obtained. Both the systems of ordinary differential equations (ODEs) and delay differential equations (DDEs) are then solved taking into account of the experimental data and physical properties of IPMC and the results are discussed and validated.**

Keywords: Active actuator, Lambert-W function, Delay-free, Stability analysis, pre-shape function

I. INTRODUCTION

Smart or active materials that can map an environmental attribute into a quantitative response i.e. can sense changes or fluctuations in the environment at the same time process the information and respond accordingly. Polymers that respond to external stimuli by changing the shape or size have been known and studied over the decades. They respond to stimuli such as an electrical field, pH, a magnetic field, and light **[1].** These responsive polymers can be collectively called active polymeric actuator or sensor depending on their functioning.

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Various types of active polymer with different controllable features exhibit change in volume or shape under a variety of stimuli. Depending on the type of actuation, these materials are broadly classified as non-electrically deformable polymers (actuated by nonelectric stimuli such as pH, light, temperature, etc. and called ceramic actuator) and electro-active polymers (EAPs) (actuated by electric input) **[2].** These types of systems always exhibit a finite time delay between the application of a stimulus and the corresponding output response of the system. In many physical and engineering systems, pure delays are often incorporated to ideally represent the effects of transmission, actuation and inertial phenomena. In biological systems, this delay may well be of the magnitude of a few hundred milliseconds duration **[3]**. Thus, time delayed systems are those where a significant time delay exists between the application of input to the system and their resulting output response. Such systems respond with an intrinsic time delay due to their working mechanism or in the components or sometimes an intended time delay is introduced for control purposes. Ordinary differential equations (ODEs) are generally constituted to describe system dynamics. Even though, they are well suited for many practical control problems, dynamical systems essentially exhibit some delays due to transmissions of signals, materials or information **[4-6]**. For example, in feedback control systems, the feedback structures physically involve propagations delays. Differential equations that incorporate time-delay factors are called delay differential equations or time-delay systems in terms of the control engineering. In practical applications, in addition to feedback controllers, sensor, actuator or digital computers are often equipped in the system construction and these components also inherently add some time-delays due to completions of their responses. Thus, the time-delays are essential features of system dynamics and therefore time-delay systems have received much attention by many researchers and developed many solution procedures and control techniques up to date **[7-9]**.

To minimize the effect of actuator delay, various compensation methods have been developed so far and applied in real-time testing. Real-time testing is a useful technique to evaluate the performance of structural systems subjected to seismic loading. Cheng et al proposed an approach to analyze actuator delay compensation using an equivalent discrete transfer function. Discrete control theory is introduced to formulate the discrete transfer function in the frequency domain and the difference equation in the time domain for an actuator delay compensation method. Three compensation methods were selected to illustrate the proposed approach. The selected compensation methods are shown to have different characteristics of amplitude and phase in their frequency response **[10]**. Delay differential equations (DDEs) have also been widely used as model for regenerative machine tool vibration. Tobias et al introduced classical chatter theories where they developed a graphical and an algebraic method to determine the onset of instability of a system with multiple degrees of freedom **[11]**.

Ionic polymer metal composites (IPMCs) are relatively a new class of EAP actuator that can serve a dual purpose in engineering or biomedical application as sensor or actuator. Though the actuating and sensing properties of IPMCs are known over a decade, some drawbacks of Nafion/Flemion based IPMC actuator limits application of the actuator. Time delay is one of the key issues as solvent water along with mobile cations takes some time to transport themselves to other side. The time delayed vibration characteristics of ionic polymer metal composite actuator (IPMC) is observed to be due to the structural flexibility of the system; which also changes with change in the applied potential difference, polymer property (type of polymer, counter ion, hydration /moisture level , i.e. water content, electrode layer (type, area, thickness), and as well as other environmental conditions. Till date, the actuation capabilities of IPMC have been studied extensively; although the time delayed characteristics and associated response analysis have received very poor attention. It has been observed by Punning et al that to use IPMC actuator in real time application, time delay factor is to be considered **[12]**. Thus, time delay analysis and an approximate closed form solution are required for studying their response during actuation. Kanno et al developed first black box model of ion conducting polymer film (ICPF) that provides a purely empirical model of the IPMC. This method though offers the least amount of insight into the fundamental mechanisms of IPMC but describes the actuator element through a series of curve fits based on experimental data **[13]**. Active vibration and noise control of distributed systems with delay were investigated by Olgac et al **[14]**.

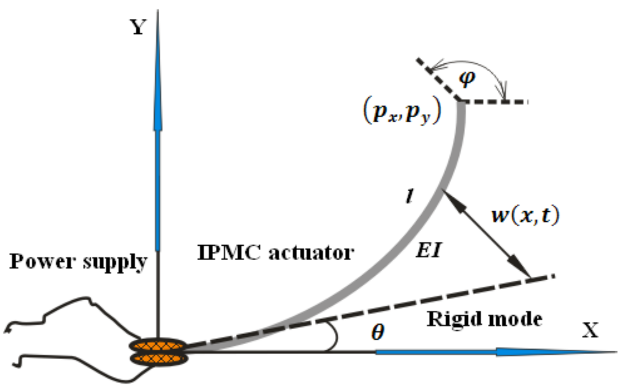
Past literatures and experimental results reveal that a finite time delay exists between the input given (on/off) to the actuator and the resulting output response and this certainly affect controlling the vibration response of the active material actuator. Further, past literatures show that very limited topics are touched down or no works have been carried out that address the time delayed vibration response and a suitable control strategy to compensate it. This, further motivate to explore the grey areas exist on time delayed response of active polymeric actuator since very limited progress has been achieved till date. Thus, it is relevant and important to study the time delayed vibration response during actuation; particularly, in this work a silver-electrode (Ag) IPMC actuator is selected to study the time delayed response under electric stimulus.

In addition, past researches suggest that IPMCs suffer low amplitude vibration during actuation where, further time delay amplify the dead-zone in response, results in additional controller tasks. The main objective of the present work is to develop a delay-model and obtaining the solution by applying Lambert-W function and study the time delayed vibration response of IPMC under electric potential. 'Time-delay' in response is quantified as the total time required to respond i.e. time taken by the cations with water molecules and solvents of ionic polymer to migrate towards cathode side after potential is switched on or off. The actuator is subjected to DC input potential and a theoretical time-delay model in state-space form is constituted assuming multi-modes excitation in fixed-free configuration applying the Hamilton's principle. Lambert-W function is then applied to obtain a closed-form solution for the transcendental characteristic equations of DDE. Both the systems of ODEs and DDEs are then solved numerically and the results i.e. transient and steady-state vibration response are studied and validated. The proposed solution method and obtained results are instrumental to develop and study the time delayed vibration characteristics of different types of smart material actuator and to choose a suitable control strategy to compensate it.

II. MODELING OF THE ACTUATOR

Extensive past research and study on actuation mechanism of IPMCs verified that IPMC actuator exhibit bending deformation while at the same time displays mechanical vibration under electric potential. This vibration coupled with time delays make it difficult to obtain a closed-form solution necessary to control the delayed response of the actuator. Thus, it is essential to derive a mathematical model that describes both the time delay features and the vibration characteristics; and a suitable solution method to solve the problem. In this work, IPMCs are first fabricated following the chemical decomposition method. Nafion membrane with an equivalent weight (EW) of 1100 g / mol and 0.183 mm thick is used as the base polymer. The fabrication process comprises multi-steps including pretreatment, adsorption, reduction and developing. In various steps, silver nitrates GR (AgNO3), ammonia solution (NH3), sodium hydroxide (NaOH), dextrose anhydrous GR (C6H12O6) are used for fabrication. Figure 1(a) shows a fabricated silver (Ag) electrode IPMC actuator; a suitable sample of size 25mm×5mm×0.2mm is prepared and tested under electric potential. The actuator is treated as flexible distributed parameter system in fixed free configuration (as Euler-Bernoulli beam) and has been analyzed and modeled. Figure 1(b) shows the schematic of working of the IPMC actuator subjected to DC electric potential at the fixed end across its thickness and subsequently tip positions are measured at the other end. Table 1 presents the tip position measured for various input voltage. Assuming pure bending, as the cross section is to be very small in comparison to the length of the IPMC, shear deformation and rotary inertia effects, if any, are neglected in the analytical modeling.

(a)



(b)

Figure 1. (a) Fabricated Ag-IPMC actuator (b) Schematic of IPMC actuator under an input voltage V

As shown in the Figure 1, the bending deflection comprise of two components i.e. rigid body displacement $(x\theta)$ and transverse displacement $w(x,t)$ due to flexibility of the structure. Equation of motion and subsequently the time delay model of the system depend both on the kinetic and potential energies and the derivation of them is clearly discussed.

Table 1. Measured tip positions of the IPMC for various input voltage for a time interval of 20s

Input voltage (V)	Tip position (p_x) (mm)	Tip position (p_y) (mm)	Tip angle (rad)
0.2	24.8	1.0	.08
0.4	23.6	2.4	.203
0.6	23	4.5	.386
0.8	22.5	7.2	.619
1.0	21.5	9.0	.792
1.2	19.5	11	1.02

A. *Kinetic Energy of the System*

Analyzing the dynamics of the system leads to finding the relationship between the flexible body coordinate 'q' and time 't'. In this case, it is important to distinguish between two aspects i.e. obtaining the closed-form equations that describe the time evolution of the generalized coordinates and second might be knowing what generalized forces need to be applied in order to realize a particular time evolution of the generalized coordinates. Particularly, in Hamiltonian method, the actuator is treated as a whole and performs the analysis after obtaining the kinetic and potential energies etc. Kinetic energy of the actuator is due to both the rigid body motion and the vibration during actuation. The transverse displacement i.e. vibration amplitude of the actuator can be expressed as,

$$w(x,t) = \sum_{i=1}^{n} \psi_i(x) q_i(t) \quad (1)$$

where, n is the number of participating modes in the approximation, ψ_i and $q_i(t)$ are the admissible function and time modulation respectively. The details of the analysis and derivation of the model can be found in the literature **[15]**. The expression for kinetic energy is obtained as:

$$T_k = \frac{1}{2} I_{tt}\dot{\theta}^2 + \sum_{i=1}^{n} I_{tq}\dot{\theta}\dot{q}_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \dot{q}_i I_{qq}\dot{\theta}\dot{q}_j \quad (2)$$

where, $I_{tt} = \rho A \frac{l^3}{3}$, $I_{tq} = \rho A \int_0^l x\psi_i(x)dx$, $I_{qq} = \rho A \int_0^l \psi_i(x) x \psi_j(x)dx$. In addition to this, as the actuator exhibits *2D* planar motion, neglecting the gravitational effect, the potential energy due to deformation of the actuator is obtained as;

$$U_p = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_i^T K_{ij} q_j \quad (3)$$

where, $K_{ij} = \int_0^l \frac{\partial^2\psi_i(x)}{\partial x^2} EI \frac{\partial^2\psi_j(x)}{\partial x^2} dx$. Further, works needed to overcome the damping of the actuator can be expressed as,

$$W_d = -\int_0^l C\dot{w}w dx = -\sum_{i=1}^{n}\sum_{j=1}^{n} \dot{q}_i^T C_{ij} q_j \quad (4)$$

where, C is the equivalent viscous damping coefficient; and is obtained as, $C_{ij} = \int_0^l \psi_i(x) C \psi_j(x)dx$. In addition to this, work done by the actuator due to applied potential V, i.e., with infinitesimal bending $\delta\theta$,

$$\delta W_b = M\delta\theta \quad (5)$$

where, M is the bending moment.

B. *Equation of Motion*

After obtaining the expressions of kinetic and potential energies and the virtual work done, the overall energy variation of the actuator equals,

$$\int_0^l (\delta T_k - \delta U_p + \delta W_d + \delta W_b)dt = 0 \quad (6)$$

Few steps of simplification and manipulation yield the equation of motion of the actuator and are of the form as,

$$I_{tt}\ddot{\theta} + \sum_{i=1}^{n} I_{tq}\ddot{q}_i = M \quad (7a)$$

$$I_{tq}\ddot{\theta} + \sum_{j=1}^{n} I_{qq}\ddot{q}_i + \sum_{j=1}^{n} C_{ij}\dot{q}_i + \sum_{j=1}^{n} K_{ij} q_i = 0 \quad (7b)$$

where, $q_i = 0, 1, 2, 3, \ldots\ldots, n.$, and $M = \frac{EI}{R}$ obtained experimentally (Table 1). The resulting '$n+1$' equations of motion can be written in terms of mass (I_m), damping (C_m) and stiffness (K_m) matrices that are in general configuration dependent, i.e

$$I_m\ddot{\Theta} + C_m\dot{\Theta} + \mathrm{K_m}\Theta = \mathrm{M_m} \tag{8}$$

For single mode approximation, i.e. $n=1$, the coefficients of equation (8) can be expressed as,

$$I_m = \begin{bmatrix} I_{tt} & I_{tq} \\ I_{tq} & I_{qq} \end{bmatrix}, C_m = \begin{bmatrix} 0 & 0 \\ 0 & C_{11} \end{bmatrix}, K_m = \begin{bmatrix} 0 & 0 \\ 0 & K_{11} \end{bmatrix}, \mathrm{M_m} = \begin{bmatrix} M \\ 0 \end{bmatrix} \text{ and } \Theta = \begin{bmatrix} \theta \\ q \end{bmatrix}$$

In state-space form, the equation (8) can be expressed as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} [0]_{2\times2} & [I]_{2\times2} \\ \left[-\frac{K_m}{I_m}\right]_{2\times2} & \left[-\frac{C_m}{I_m}\right]_{2\times2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} [0]_{2\times1} \\ \left[\frac{M_m}{I_m}\right]_{2\times1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{9}$$

where, $x_1 = \begin{bmatrix} \theta \\ q \end{bmatrix}$ and $x_2 = \begin{bmatrix} \dot{\theta} \\ \dot{q} \end{bmatrix}$.

In order to derive the delay model of the actuator, let us assume that the actuator responds after a time interval T of potential input. Thus, incorporating the delay, the generalized delay differential equation (DDE) of the actuator in state-space form can be expressed as:

$$\begin{aligned} \dot{x} + Ax(t-T) + Bx(t) = 0 \\ x(t) = \varphi, \quad t \in [-T, 0] \end{aligned} \tag{10}$$

where, $\varphi(t)$ is (4×1) initial pre-shape function and is defined in the interval of $(-T, 0)$. Comparing (9) and (10), delay matrix 'A' and system matrix 'B' are obtained as:

$$A = \begin{bmatrix} [0]_{2\times2} & [0]_{2\times2} \\ \left[-\frac{K}{I_m}\right]_{2\times2} & [0]_{2\times2} \end{bmatrix} \text{ and } B = \begin{bmatrix} [0]_{2\times2} & [-1]_{2\times2} \\ \left[\frac{K_m}{I_m}\right]_{2\times2} & \left[\frac{C_m}{I_m}\right]_{2\times2} \end{bmatrix}$$

where, K is the spring constant of IPMC. Both the system matrices depend on the inertia, stiffness, damping of the actuator and the bending moment developed. The interval $(-T, 0)$ can be termed as pre-interval while $\varphi(t)$ is called pre-shape function and is part of the configuration of $x(t)$ in the pre-interval.

III. SOLUTION OF DDE VIA LAMBERT FUNCTION

The main advantage of obtaining solution of DDEs applying the Lambert W function is that the solution obtained by this method can be compared to the general solution obtained by solving ordinary differential equation (ODE). Further, in matrix method, the state transition matrix in ODE's can be generalized to DDE using the concept of Lambert function. To find out solution of the equation (10), let us try to begin with the candidate solution of the form,

$$x(t) = e^{St}x_0 \tag{11}$$

Substituting (11) into (10) and finding that $e^{St} \neq 0$ yields;

$$s + Ae^{-ST} + B = 0 \tag{12}$$

After few steps of simplification and manipulation, equation (12) yields:

$$(S+B)Te^{ST}e^{BT} = -ATe^{BT} \tag{13}$$

In certain case, where the matrices A and B do not commute, i.e. $(S+B)Te^{(S+B)T} \neq (S+B)Te^{ST}e^{BT}$. The problem is solved by introducing an unknown matrix 'V' that satisfies the condition as:

$$(S+B)Te^{(S+B)T} = -ATV \tag{14}$$

To obtain the solution, equation (14) is expressed via Lambert function that satisfies the following equality i.e.

$$W(V)e^{W(V)} = V \tag{15}$$

where, V is a $n \times n$ matrix variable. Thus, comparing (14) and (15), we find that

$$(S+B)T = W(-ATV) \tag{16}$$

Solving for S, the expression is obtained as,

$$S = \frac{1}{T}W(-ATV) - B \tag{17}$$

The unknown matrix V can be found out substituting (17) into (12), which yields the following relationship

$$W(-ATV)e^{W(-ATV)-B} = -AT \tag{18}$$

Equation (18) is solved to get the solution of the transcendental characteristic equation (12). This shows that the equation (11) is solution of the system defined in (10), where the solution 'S' is defined in (17) and can be expressed as **[3, 17]**.

$$x(t) = x_0 e^{\left[\frac{1}{T}W(-ATV)-B\right]t} \tag{19}$$

The general solution of the first order matrix delay differential equation (10) is thus linear combination of all the solutions i.e.

$$X_r(t) = \sum_{r=-\alpha}^{\alpha} H_r\, e^{\left[\frac{1}{T}W_r(-ATG_r)-B\right]t}, r = 0, \pm1, \pm2, .. \tag{20}$$

where, H_r is the coefficient, $(n \times 1)$ matrix variables for each branch of the Lambert function. The coefficient H_r is found out satisfying the given pre-shape function $\varphi(t)$. The pre-shape function $\varphi(t)$ at $t = 0$ for 'r' number of branches can be expressed as,

$$\varphi(0) = \cdots + h_{-r}x_{-r}(0) + h_{-(r-1)}x_{-(r-1)}(0) + \cdots + h_{-1}x_{-1}(0) + h_0x_0(0) + h_1x_1(0) + \cdots \tag{21}$$

The coefficient 'h' expresses the $(n \times 1)$ matrix variables and can be evaluated separately for each branch of the Lambert function. Much more on the solution technique can be found in the literatures **[3, 16-17].** Further, stability of the system for principal branch of the Lambert function can be obtained by satisfying the following expression, i.e.,

$$\left|\frac{1}{T}W(-ATV) - B\right| \leq \pi \tag{22}$$

This further yield,

$$\pi - \left|\frac{1}{T}W(-ATV) - B\right| \geq 0 \tag{23}$$

IV. RESULTS & DISCUSSIONS

Results are obtained through numerical simulation taking into account of the experimental data and properties as given in the Table 1 and Table 2 and are discussed. A program is developed in MATLAB to solve the differential equation in state-space form (9) & (10) for obtaining the vibration response for various input voltage. The tip deflections data are measured after 30s for each input voltage. Physical properties and size of IPMC actuator used for simulation are given in Table 2. The damping coefficient is obtained as $C = 2\xi\omega_n$, where; first three modal frequency of vibration is obtained as 2.0174 rad/s, 12.6442 rad/s and 35.4044 rad/s. The damping factor $\xi = 0.0197$ of the IPMC actuator is obtained experimentally. The solution for unknown matrix V_r is obtained numerically using the 'fsolve' command. Note that V_r has unique solution for each branch of the Lambert function in matrix form. The roots of the solution in the equation (20) can be obtained using method like converting the matrix $'ATV'$ into diagonalized form. The characteristic roots are therefore the Eigen-values of the matrix $W(-ATV)$ and they describe the stability of the vector delay differential equation. In this case, Jordon's similarity transformation is used for decomposing the matrix and the value of $W_r(ATV_r)$ is obtained. The pre-shape function is defined in the pre-interval $(-1,0)$ as: $\varphi(t) = [0.5;\ 0.05; 0; 0\ ]$. The results are obtained only for the principal branch of the Lambert W function i.e. $W_0(x)$, other branches of the function do not yield solution. Further, equation (9) is solved as ordinary differential equation (delay-free) using RK4 method and the results are validated with the solution obtained by solving delayed equation (10). The initial search matrix tried for obtaining the V matrix is given as: $V_i = [I]_{4\times4}$. The equations are solved and the coefficients for the principal branch are obtained and are given in the Table 3.

Table 2. Physical properties and size of IPMC actuator

Property	IPMC
Elastic modulus (*E*)	0.081880 GPa
Length (*l*)	0.025 m
Width (*w*)	0.005 m
Thickness (*h*)	0.0002 m
Density (ρ)	2125 kg/m^3
Spring constant (*K*)	1.3646×10^{-5} Nm
Mass (*m*)	5.3125×10^{-5} kg

Table 3 gives the coefficients obtained for various input voltage for the principal branch of the Lambert function. The characteristics of solution highly depend on the delay matrix A, the delay time T and the pre-shape function $\varphi(t)$.

Table 3. Coefficients obtained for the principal branch of the Lambert-W function with a delay $T = 1s$

Input (V)	Coefficient $[H_0] = [h_0{}^1 \quad h_0{}^2 \quad h_0{}^3 \quad h_0{}^4]$
0.6	0.2793 + 0.1434i, 0.2793 - 0.1434i, 1.4021e-005 +5.9935e-023i, 0.0712 - 0.0000i
1.2	0.1934 - 0.2131i, 0.3719 + 0.2205i, -3.8940e-005 -1.2165e-005i, -0.0739 - 0.0142i

A. Time Delayed Response

The numerical simulation is performed only for the principal branch of the Lambert function; other branches do not yield any solution. Figure 2 and Figure 3 show the vibration response of the actuator for an input of 0.6V and 1.2V respectively. Figure 2 shows the response with a delay of $T = 1s$ while Figure 3 shows the response without delay i.e. obtained after solving the ordinary differential equation (9) in state-space form. The initial condition is maintained same as the pre-shape function $\varphi(t)$. It is observed that vibration response with delay (T = 1s) closely matches with the results of the system without delay (solving ODEs). This certainly validates the results and accuracy of the delay model that is developed using the Lambert-W function.

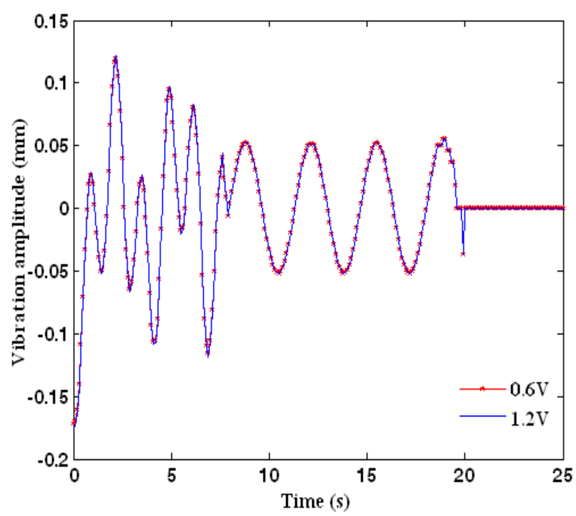


Figure 2. Vibration response of the actuator combining primary two modes with a delay T=1s (DDEs)

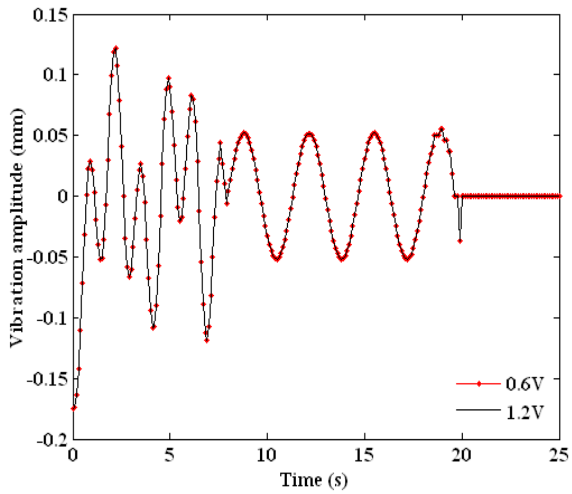


Figure 3. Vibration response of the actuator combining primary two modes without delay (ODEs) i.e. T == 0s

These results show over a time period of 25s; while experimental data are measured over 30s (enough time is allowed to reach steady-state and settle for back relaxation). It is observed that the actuator reaches steady-state condition and settle by 20s after the voltage is applied. Figures 4 and 5 are showing the vibration response combining primary three modes of the actuator. As anticipated, there is minute change in the vibration amplitude, although as expected both the results validate the delay model developed using Lambert W-function. It is further observed that though the input voltage increases from 0.6V to 1.2V, the effectiveness of the actuator remains consistent exhibiting similar types of response in both the cases.

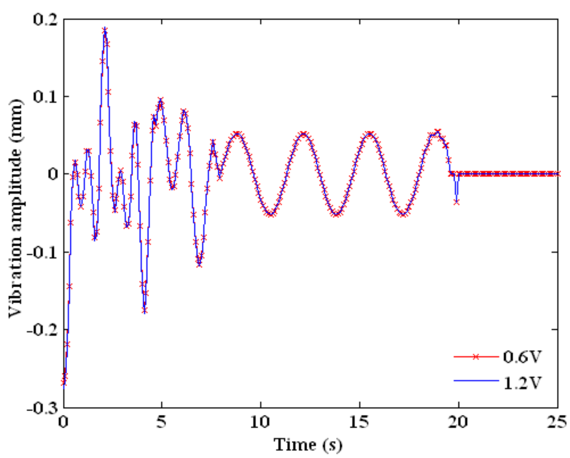


Figure 4. Vibration response of the actuator combing primary three modes with a delay of T=1s (DDEs)

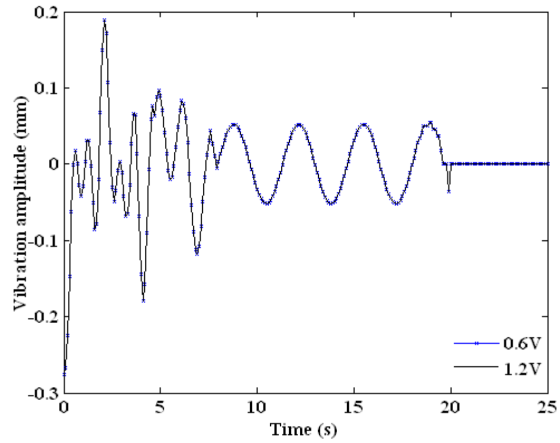


Figure 5. Delay free (T=0s) vibration response of the actuator combining primary three modes (ODEs)

Figure 6 shows the rate of change of transverse displacement of the actuator with a delay of 1s while Figure 7 shows the response without delay (0s) combining first two modes of the actuator. It is observed that with delay, the rate of change of transverse displacement increases (though negligible in magnitude) as the input voltage increases, whereas, without delay (ODEs) the magnitude remains to be closely matching. However, it is shown that vibration response i.e. transverse displacement for the both cases remains identical with same input voltage.

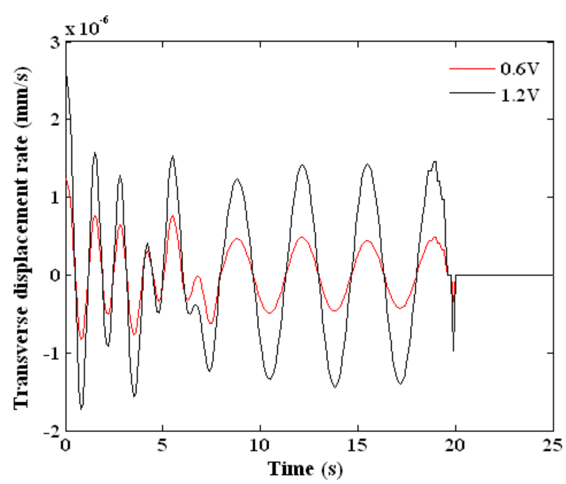
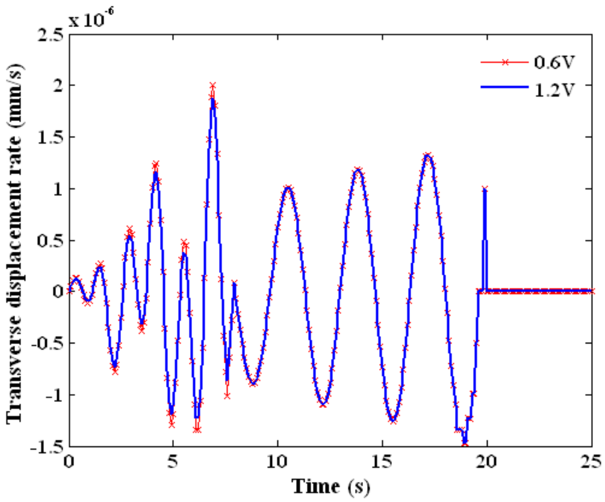


Figure 6. Rate of change of transverse displacement of the actuator combing primary two modes with a delay T=1s



(b)

Figure 7. Tip velocity of the actuator combing first two modes i.e. response without delay T=0s

It is observed that irrespective of delay to the actuator response, the rate of change of transverse displacement of the actuator remains alike. It is further shown that with increase in input voltage the magnitude of the transverse displacement rate increases in case of DDE while it remains similar for ODE. As the response of the actuator, significantly depends on the pre-shape function $\varphi(t)$ and delay matrix A, it is well understood that system response changes considerably with change of these parameters. Figure 8 shows the vibration response of the actuator with a delay $T = 0.5s$,where, pre-shape function is defined in the pre-interval $(-0.5,0)$ as, $\varphi(t) = [0.08;\ 0.02; 0; 0\ ]$. In this case, the coefficients obtained from pre-shape function changes for each input voltage, where the position in the interval $(-T, 0)$ is expressed as $x(t) = x_1(t)$. This result clearly validates that with change in delay to the system and pre-shape function, the vibration response of the actuator changes significantly.

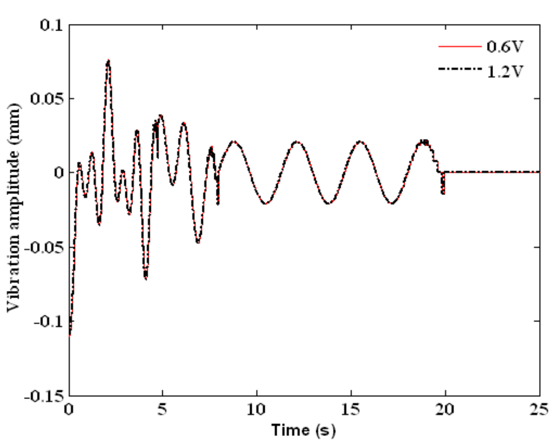


Figure 8. Vibration response of the actuator combining primary 3 modes with delay T=0.5s (DDEs)

V. Conclusion

In this article, a new approach for solving the time delayed vibration response of an active material actuator using Lambert W function has been proposed. An analytical model of IPMC actuator has been developed using experimental data and following the Hamilton's principle. Lambert W function is then applied to obtain the closed form solution of the transcendental characteristic equation of the DDE. The advantage of the proposed approach is in obtaining the closed form solution of the delay equation in a composite form similar to ordinary differential equation. Both the DDEs and ODEs are solved and the results are discussed and the delay model is validated. This suitability enables us to apply the presented solution technique to solve more complex time delayed vibration problem of active material actuator system. The proposed method for obtaining the solution is useful in deriving the control strategy and compensation for delay of active material actuator.

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