Polynomials with Integers Coefficients with Integers Critical and Inflection Points

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Finding integer polynomials (polynomials with integer coefficient) with integer critical and inflection points are key to find good examples in a calculus class. But finding such polynomials are sometimes challenging and time consuming and usually text book does not provide enough such examples. We provide a very simple code in 'Mathematica" to generate polynomials of degree 4 with integer coefficient which has 3 integer critical points and 2 integer inflection points. The code can be generalize or extended to create higher degree polynomial with similar characteristic.

Key Word: "Integer Polynomial"; "Critical Point", "Inflection Point", "Calculus"

The reader just need to copy and paste the following code in the "Mathematica" and run the code. To know more about this program and to download this into your computer please refer to the website www.wolfram.com. Also introductory user can read the help in (Purdue University) (Wolfram Mathematica). Some other interested use of this software, including classroom demonstration and student summer research can be found in (Image Compression) (Prince and Ruslan, Developing and Teaching an Honors Calculus Course in a Community College) and (Prince, Flipping Over Math: Computer Based Homework and Quizzes for Non-Traditional Students). The program will ask for two input from the reader – an odd positive integer "n" and a positive integer "k" and the output will produce a polynomial of degree 4 with integer coefficient which has 2 integer inflection points and 3 integer critical points. Everything written in bold font is the code.

```
ClearAll;

(* choose n to be any odd positive integer and k to be any positive integer but make sure k and n are different. *)

n=3;
k=2;
t=Solve[a-3b    3^n&&3a-b    k^2,{a,b}];


(* The following will be the second derivative and the first derivative of the function we are looking for...*)
aa[x_]:=(x/8-a)(x/8-b)/.t[[1]];
vv[x_]:=Integrate[aa[x],x]/.t[[1]];
(* The correponding criticals points will be....*)

criticalpoints= Solve[vv[x]    0,x];
```

```
Print["The critical points are"]
criticalpoints

(* the points where second derivative is 0 will be
of course a and b *)

inflectionpoints= Solve[aa[x]    0,x];
Print["The inflection points are"]
inflectionpoints

(* the polynomial that are looking for ....*)

pp[x_]:=Integrate[vv[x],x];
Print[" the actual polynomial is "]
Denominator[Together[pp[x]]]Together[pp[x]]
```

For different values of "n" and "k", the code will produce different polynomials. The output also lists all the critical and inflection points of the polynomial. Some examples are given below: (we list the polynomials that we get by choosing different value of "n" and "k"):

$$-270x^2 - 8x^3 + x^4$$
$$-48x^3 + x^4$$
$$1890x^2 - 104x^3 + x^4$$
$$6912x^2 - 176x^3 + x^4$$
$$936x^2 + 64x^3 + x^4$$
$$11520x^2 + 208x^3 + x^4$$
$$6930x^2 + 184x^3 + x^4$$

You can easily generate as many as you want and they all have integer critical points and integer inflection points. Of course, you can add any constant term to the polynomial as it does not affect the derivative.

You can change the code to produce higher degree polynomials with same characteristic but we have not demonstrated this in the current paper and leave it up to the reader to experiment with the code.

References

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