Urysohn's Lemma in Soft Topological Spaces

Mrudula Ravindran and Remya.P.B.

Abstract: Soft set theory is one of the emerging branches of mathematics that could deal with parameterization inadequacy and vagueness that arises in most of the problem solving methods. It is introduced in 1999 by the Russian mathematician D.Molodtsov with its rich potential applications in divergent directions such as stability and regularization, game theory, operations research, soft analysis etc. Further research works produced so many definitions, results and practical applications. It got some stability only after the introduction of soft topology in 2011. It is found that most of the results in general topology are true in soft topology. In this paper it is proved that Urysohn's lemma is valid in soft topological spaces also.

Keywords: Soft continuity, Soft interior point, Soft point, Soft neighborhood, Soft normal space, Soft topology.

I. INTRODUCTION

"Muhammad shabir and Munazza naz [12] (Pakistan)" and Cagman et al., [2] initiated the study of soft topology and soft topological spaces independently. They defined soft topology on the collection τ of soft sets over U with a fixed set of parameters. Besides studying the basic properties of soft topological spaces they have also introduced soft separation axioms. On the other hand Cagman et al., introduced a soft topology on a soft set and defined a soft topological space.

Babitha and Sunil [1] introduced soft function. Kharal and Ahmad [9] gave the concept of soft mapping. The concept of the soft point is expressed by different approaches. Cigdem gunduz et al., [3] introduced soft continuous mappings in 2013, which are defined over an initial universe with a fixed set of parameters. The continuity of mappings of soft topological spaces has defined and its properties have investigated. Soft open mappings, soft closed mappings, soft homeomorphisms are all studied with examples. I.Zorlutuna et al., [15] introduced the concept of soft neighborhood, soft continuity etc. All these works made foundation stones for this paper.

Mrudula Ravindran, H.O.D, Dept. of Mathematics, C.M.S.College of Science & Commerce, Coimbatore, Tamil Nadu, India. Email:mrudularavindran@gmail.com
Remya.P.B, Research Scholar, C.M.S.College of Science & Commerce, Coimbatore, T.N. E mail: thulasi3krish@gmail.com

II. PRELIMINARIES

Definition 1[11]: Let U be an initial universe, E be a set of parameters and P (U) be the power set of U. A pair (F, A) is called a *soft set* over U where A⊆E and F: A⟶P (U) is a set-valued mapping.
Remark 1[11]: A soft set over U is a parameterized family of subsets of the universe U.
Remark 2[11]: For particular e∈A, F (e) may be considered as the set of e - approximate elements of the soft set (F, A).
Remark 3[5]: A soft set (F, A) can be viewed as: (F, A) = {F (e) /e ∈ A}.
Remark 4[15]: In order to efficiently discuss usually soft sets over a universe U are considered with a fixed set of parameters A. The family of these soft sets are denoted by $SS(U)_A$ while the set of all soft sets over U is denoted by S(U).
Remark 5 [11]: Soft set (F, A) can be denoted as F_A.
Definition 2[15]: The *complement of a soft set* (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$. F^c: A⟶P (U) is a mapping given by $F^c(e)$ = U—F (e), for each e∈A. Here F^c is called soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.
Remark 5[15]: $(Ø, A)^c = (U, A)$ and $(U, A)^c = (Ø, A)$.
Definition 3[6]: A soft set (F, A) over U is said to be *null - soft set* if for all e∈A, F (e) = Ø. It is denoted by (Ø, A) or $Ø_A$.
Definition 4[6]: A soft set (F, A) over U is said to be an *absolute soft set* if for all e∈A, F (e) = U. It is denoted by (U, A) or U_A.
Definition 5[8]: Let U is an initial universe and τ is the collection of soft sets over U then τ is called as *soft topology* on U if

i. null and absolute soft sets belongs to τ
ii. arbitrary union of soft sets in τ belongs to τ
iii. finite intersection of soft sets in τ belongs to τ

Definition 6[8]: The triplet (U, A, τ) is called as *soft topological space* or *soft space* over U.

Definition 7[8]: Let (U, A, τ) be a soft topological space over U. Then the members of τ are said to be *soft open sets* in (U, A).

Definition 8[8]: Soft set (F, A) over U is said to be *soft closed* in (U, A), if its relative complement belongs to τ.

Proposition 1[7]: The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it

Definition 8[11]: The soft set (F, A)∈SS(U)$_A$ is called a *soft point* in (U, A) denoted by $e_{(F,A)}$ or e_F if for the element e∈A, F(e) ≠ Ø and F(e') = Ø for all e'∈ A−{e}.

Remark 6: Classical set theoretical operations with ~ symbol above are used to distinguish between classical and soft set operations.

Definition 9[11]: For two soft sets (F, A) and (G, B) over a common universe U, (F, A) is a soft subset of (G, B) if (i) A⊂B and (ii) for every e∈A, F (e) and G (e) are identical approximations. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A).

Definition 10[11]: The soft point e_F is said to be in the soft set (G, A) denoted by e_F $\tilde{\in}$ (G, A), if F (e) $\tilde{\subseteq}$G (e) for the element e $\tilde{\in}$A.

Definition 11[15]: Let $e_F$$\tilde{\in}$ (U, A) and (G, A) $\tilde{\subseteq}$(U, A). If e_F $\tilde{\in}$ (G, A), then e_F $\tilde{\notin}$ (G, A)^c

Definition 12[11]: Let (U, A, τ) be a soft topological space and let (G, A) be a soft set over U. The soft point e_F $\tilde{\in}$ (U, A) is called a soft interior point of a soft set (G, A) if there exists a soft set (H, A) such that e_F $\tilde{\in}$ (H, A) $\tilde{\subseteq}$ (G, A).

Proposition 2[11]: Let e_F $\tilde{\in}$ (U, A) for all e∈A and (G, A) be a soft open set in a soft space (U, A, τ). Then "Every soft point e_F is a soft interior point".

Definition 13[11]: A soft set (G, A) in (U, A, τ) is called a soft neighborhood (briefly: nbd) of the soft point e_F $\tilde{\in}$ (U, A) if there exists a soft open set (H, A) such that e_F $\tilde{\in}$ (H, A) $\tilde{\subseteq}$ (G, A). The neighborhood system of a soft point e_F denoted by Nτ (e_F) is the family of all its neighborhoods.

Definition 14[11]: A soft set (G, A) in (U, A, τ) is called a soft neighborhood (briefly: nbd) of the soft set (F, A) if there exists a soft open set (H, A) such that (F, A) $\tilde{\subseteq}$ (H, A) $\tilde{\subseteq}$ (G, A).

Proposition 3[9]: Soft set (F, A) over U is soft open ⇔ (F, A) is a soft nbd of each of its soft elements.

Proposition 4[4]: Let (F, A) and (G, A) be soft sets in SS(U)$_A$.Then the following are true.

1. (F, A) $\tilde{\subseteq}$ (G, A) ⇔ (F, A)$\tilde{\cap}$(G, A)= (F, A)
2. (F, A) $\tilde{\subseteq}$ (G, A) ⇔ (F, A)$\tilde{\cup}$(G, A)= (G, A).

Proposition 5[2]: Neighborhood system Nτ (e_F) at e_F in soft space (U, A, τ) has the following properties: (1) if (G, A) ∈ Nτ (e_F), then e_F $\tilde{\in}$ (G, A) (2) if (G, A) ∈ Nτ (e_F) and (G, A)$\tilde{\subseteq}$ (H, A) then (H, A) ∈ Nτ (e_F).
(3) if (G, A), (H, A)∈ Nτ (e_F) then (G, A)$\tilde{\cap}$(H, A)∈ Nτ (e_F)
(4) if (G, A)∈ Nτ (e_F) then there is a (M, A)∈ Nτ (e_F) such that (G, A)∈ Nτ (e'_H) for each e'_H $\tilde{\in}$ (M, A). *Theorem 1[4]:* A soft set (G, A) is soft open ⇔ (G, A) is a soft neighborhood of (F, A) for each soft set (F, A) contained in (G, A).

Proposition 6[15]: Let (F, A), (G, A) ∈ SS(U)$_A$. Then following are true

(1) If (F,A)$\tilde{\cap}$(G,A) = (Ø, A) then (F,A)$\tilde{\nsubseteq}$ (G,A)^c (2) (F,A) $\tilde{\subseteq}$ (G,A) ⇔ (G,A)^c$\tilde{\subseteq}$ (F,A)^c

Definition 15[12]: Let (U, A, τ) be a soft topological space over U. Let (F, A) and (G, A) be two soft closed sets over U such that (F, A) $\tilde{\cap}$(G, A) = (Ø, A). If there exist soft open sets (F_1, A) and (F_2, A) such that (1) (F, A) $\tilde{\subseteq}$(F_1, A) (2) (G, A) $\tilde{\subseteq}$(F_2, A) and (3) (F, A)$\tilde{\cap}$ (G, A) = (Ø, A). Then (U, A, τ) is called a *soft normal space*.

Theorem 2[13]: Let (U, A, τ) be a soft space over U. Let (F, A) and (G, A) are soft sets over U. Then (1) (F, A)$\tilde{\subseteq}$ $\overline{(F,A)}$ (2) (F, A) is a soft closed set ⇔ (F, A) = $\overline{(F,A)}$ (3) (F, A)$\tilde{\subset}$ (G, A) ⇒$\overline{(F,A)}$ $\tilde{\subset}$ $\overline{(G,A)}$

Definition 16[3]: Let (U, A, τ) and (V, A, τ') be two soft topological spaces. f :(U, A, τ) → (V, A, τ') be a mapping. For each soft neighborhood (H, A) of (f (e_x), A), if there exists a soft neighborhood (F, A) of (e_x , A) such that f ((F, A))⊂(H, A). Then f is said to be soft continuous mapping at (e_x , A).

If f is soft continuous mapping for all (e_x, A), f is called *soft continuous mapping.*

Theorem 3[3]: Let (U, A, τ) and (V, A, τ') be two soft topological spaces. f: (U, A, τ) → (V, A, τ') be a mapping. Then following conditions are equivalent:

(1) f :(U, A, τ) → (V, A, τ') is a soft continuous mapping (2) For each soft open set (G, A) over V, f^{-1}((G, A)) is a soft open set over U.

Proposition 7[13]: Let U, V be two non-empty sets and f: U →V be a mapping. If (F, A) ∈ S (U) then

(i) (F, A)$\tilde{\subseteq}$ f^{-1} f (F, A)
(ii) f^{-1} f (F, A) = (F, A) if f is injective.

Proposition 8[13]: Let U, V be two non-empty subsets and f: U →V be a mapping. If (G_1, A), (G_2, A) ∈ S (V) then

(i) (G_1, A)$\tilde{\subseteq}$ (G_2, A) ⇒ f^{-1}(G_1, A)$\tilde{\subseteq}$ f^{-1}(G_2, A) (ii) f^{-1}[(G_1, A)$\tilde{\cup}$ (G_2, A)] = f^{-1}(G_1, A) $\tilde{\cup}$ f^{-1}(G_2, A) (iii) f^{-1}[(G_1,A)$\tilde{\cap}$ (G_2,A)] = f^{-1}(G_1,A)$\tilde{\cap}$ f^{-1}(G_2,A)

III. URYSOHN'S LEMMA IN SOFT TOPOLOGICAL SPACES

Urysohn's lemma in general topology asserts that in every topological space which is normal, two closed subsets may be separated by a real-valued function. Here it is tried to find out whether this lemma is true in soft topological spaces. The concepts of soft neighborhood and soft continuity have formed the base for this.

Lemma 1: Let (U, A, τ) be a soft topological space. Let one-point sets in (U, A, τ) be soft closed.

(U, A, τ) is soft normal ⟺ given a soft closed set (F, A) and a soft open set containing (F, A), there exists a soft open set (H_1, A) such that (F, A) $\widetilde{\subseteq}$ (H_1, A)$\widetilde{\subseteq}$ $\overline{(H1,A)}$ $\widetilde{\subseteq}$ (U, A).

Proof: Let (U, A, τ) is a soft normal space and (U, A) be a soft open set containing the soft closed set (F, A) in (U, A, τ). (F, A) is soft closed⟹ (U, A, τ) – (F, A) is soft open. (U, A) is soft open⟹ (U, A, τ) – (U, A) is soft closed. (F, A) $\widetilde{\cap}$ [(U, A, τ) – (U, A)] = [(F, A) $\widetilde{\cap}$ (U, A, τ)] – [(F, A) $\widetilde{\cap}$ (U, A)] = (F, A) – (F, A) = (Ø, A). ∴ (F, A) and (U, A, τ) – (U, A) are disjoint soft closed sets in (U, A, τ).

Using soft normality we can find a pair of disjoint soft open sets (H_1, A), (H_2, A) in (U, A, τ) such that (F, A) $\widetilde{\subseteq}$ (H_1, A) and (U, A, τ) — (U, A)$\widetilde{\subseteq}$ (H_2, A). (H_1, A)$\widetilde{\cap}$(H_2, A) = (Ø, A) ⟹ (H_1, A) $\widetilde{\subseteq}$(U, A, τ) — (H_2, A) ⟹$\overline{(H1,A)}$ $\widetilde{\subseteq}$=(U, A, τ)—(H_2, A)⟹$\overline{(H1,A)}$ $\widetilde{\subseteq}$ (U, A).Since (H_1, A) soft open ⟹(H_1, A) $\widetilde{\subseteq}$ $\overline{(H1,A)}$. ∴ (F, A) $\widetilde{\subseteq}$ (H_1, A)$\widetilde{\subseteq}$ $\overline{(H1,A)}$ $\widetilde{\subseteq}$ (U, A).

Converse: Let (C_1, A), (C_2, A) be a pair of disjoint soft closed sets in the soft topological space (U, A, τ). ∴ (C_1, A) is contained in the soft open set (U, A, τ) — (C_2, A). By hypothesis, there exists another soft open set say (H_1, A) such that (C_1, A)$\widetilde{\subseteq}$(H_1, A)$\widetilde{\subseteq}$ $\overline{(H1,A)}$ $\widetilde{\subset}$(U, A, τ) – (C_2, A).

Let (U, A,τ) – $\overline{(H1,A)}$ = (G_2, A). Here $\overline{(H1,A)}$ $\widetilde{\subseteq}$ (U, A, τ) – (C_2, A) ⟹ (C_2, A) $\widetilde{\subseteq}$ (U, A, τ) —$\overline{(H1,A)}$, which is soft open. (H_1, A) and (G_2, A) are two soft open sets containing (C_1, A) and (C_2, A) respectively. (H_1, A) $\widetilde{\cap}$ (C_2, A) = (H_1, A) $\widetilde{\cap}$ [(U, A, τ) —$\overline{(H1,A)}$] = [(H_1, A) $\widetilde{\cap}$(U, A, τ)] — [(H_1, A)$\widetilde{\cap}$ $\overline{(H1,A)}$] = (Ø, A).

Lemma 2 (Urysohn's lemma): Let (U, A, τ) be a soft normal space; let (F, A) and (G, B) be disjoint soft closed subsets of (U, A, τ). Let [a, b] be a closed interval in the real line. Then there exists a soft continuous map f: (U, A, τ) ⟶ [a, b] such that: f (e_x) = a for every x∈ (F, A) and f (e_x) = b for every x∈ (G, B). Converse holds.

Proof: Instead of [a, b] the lemma is proved for [0, 1] The general case follows. Let D be the set of rational numbers in [0, 1]. Arrange D in some order. Let it be {1, 0, ½, 1/3, 2/3, ¼, ¾, 1/5, 2/5 …}. It is known that set of rational numbers are uncountable ⟹ the set D is uncountable. Define, for each p∈D a soft open set (U_p, A) of the soft normal space (U, A, τ). It is defined in such a manner that if p, q∈D with p < q then $\overline{(Up,A)}$ $\widetilde{\subseteq}$ (U_q, A).

Construct a sequence of soft open sets in (U, A, τ) as follows. First define (U_1, A) = (U, A, τ) — (G, B). Here (F, A) is a soft closed set contained in the soft open set (U_1, A). Using soft normality of (U, A, τ) and by lemma1 there must exist a soft open set which contains the soft closed set (F, A) and its closure is contained in (U_1, A). Let this soft open set be (U_0, A). In general let D_n denote the set consisting of the first n rational numbers in the sequence.

By assumption (U_p, A) is defined for all rational numbers p∈D with the condition that if p < q ⟹ $\overline{(Up,A)}$ $\widetilde{\subset}$ (U_q, A) ⟶ (∗) Define (U_r, A) where r is the next rational number in the sequence. Consider the set D_{n+1} = D_n∪{r}. It is a finite subset of the interval [0, 1]. In a finite simply ordered set every element has an immediate predecessor and an immediate successor. Let the immediate predecessor of r be p and immediate successor of r be q. The sets (U_p, A) and (U_q, A) are already defined and by induction hypothesis $\overline{(Up,A)}$ $\widetilde{\subset}$ (U_q, A). Here (U_q, A) is a soft open set containing a soft closed set $\overline{(Up,A)}$. Using lemma 1 there must exist a soft open set containing $\overline{(Up,A)}$ and its closure is contained in (U_q, A). Let this soft open set be (U_r, A).

∴ $\overline{(Up,A)}$ $\widetilde{\subset}$ (U_r, A)$\widetilde{\subset}$ $\overline{(Ur,A)}$ $\widetilde{\subset}$(U_q, A). It can be concluded that (∗) holds for every pair of elements D_{n+1}. If both the elements lie in D_n then (∗) holds by induction hypothesis. Let r, s be such a pair from D_n. Then either s ⩽ p or s ⩾ q. So it will lead to $\overline{(Us,A)}$ $\widetilde{\subset}$ $\overline{(Up,A)}$ $\widetilde{\subset}$ (U_r, A) and $\overline{(Ur,A)}$ $\widetilde{\subset}$(U_q, A)$\widetilde{\subseteq}$(U_s, A) respectively.

Thus for every pair of elements of D_{n+1}, relation (∗) holds. By mathematical induction (U_p, A) is defined for every p∈D. Extend this definition to all rational numbers p∈R by defining

(U_p, A) = (Ø, A); p < 0

(U, A); p > 1 ⟶ (#)

The relation (∗) is still true for any pair of rational numbers with p < q. Let $e_x$$\widetilde{\in}$ (U, A, τ). Define Q (e_x) = {p/ e_x $\widetilde{\in}$ (U_p, A)}. From (#),

Q (e_x) = Ø ; p < 0

Z^+ — {1}; p > 1

⟹ Q (e_x) is bounded below and its g.l.b. is a point in [0, 1] say f (e_x). ∴ f (e_x) = g.l.b. Q(e_x) = inf Q(e_x).

Case (i): $e_x$$\widetilde{\in}$ (F, A) then $e_x$$\widetilde{\in}$ (U_p, A) for every p ≥ 0 ⟹ Q(e_x) ={p/p$\widetilde{\in}$ $Z^+$$\widetilde{\cup}${0}}⟹ f($e_x$) = 0.

Case (ii): $e_x$$\widetilde{\in}$ (G, B) then $e_x$$\widetilde{\notin}$($U_p$, A), p ≤ 1; ⟹$e_x$$\widetilde{\notin}$ (U, A), p>1⟹Q(e_x)={p/p$\widetilde{\in}$ Z^+—{1}} ⟹ f(e_x) = 1.

For proving f : (U, A, τ) ⟶ [a, b] is continuous it is enough to prove the below elementary facts first:

(1) $e_x$$\widetilde{\in}$ $\overline{(Ur,A)}$ ⟹ f(e_x) ≤ r
(2) $e_x$$\widetilde{\notin}$($U_r$, A) ⟹ f($e_x$) ≥ r

Let $e_x$$\widetilde{\in}$ $\overline{(Ur,A)}$ ⟹ $e_x$$\widetilde{\in}$ (U_s, A) for every s > r since $\overline{(Ur,A)}$ $\widetilde{\subset}$ (U_s, A); if r < s ⟹ Q(e_x) contains all rational numbers greater than r ⟹ f(e_x) ≤ r. For proving (2) let $e_x$$\widetilde{\notin}$ (U_r, A)⟹ $e_x$$\widetilde{\notin}$ (U_s, A) for any s < r ⟹ Q(e_x) does not contain any rational number less than r ⟹ f(e_x) ≥ r. For proving soft continuity suppose e_{x0} $\tilde{\notin}$ (U, A, τ) and f (e_{x0}) $\tilde{\in}$ (c, d) $\tilde{\subseteq}$ R. Find a neighborhood (U, A) of e_{x0} such that f (U, A)$\tilde{\subset}$ (c, d). Assert that the soft open set (U, A)=(U_q, A)—$\overline{(Up, A)}$ is the soft neighborhood of $e_{x0.}$ Note $e_{x0}\tilde{\in}$ (U, A)

f (e_{x0}) < q ⟹ f (e_{x0}) $\tilde{\in}$ (U_q, A).
f (e_{x0}) > p ⟹ f (e_{x0}) $\tilde{\notin}$ $\overline{(Up, A)}$.

Let $e_x\tilde{\in}$ (U, A) ⟹ $e_x\tilde{\in}$ (U_q, A)$\tilde{\subset}$ $\overline{(U, A)}$ ⟹ f(e_x)⩽q. $e_x\tilde{\notin}$ (U_p, A) and f (e_x)⩾p. Thus f (e_x)$\tilde{\in}$ [p, q]$\tilde{\subset}$(c, d) as desired, proves the continuity of the function f : (U, A, τ) ⟶ [a, b].

Converse: Let R = {e_x∈ R: 0⩽e_x⩽1}.Let (U, A, τ) be a soft topological space and let given a pair of disjoint soft closed sets (F, A) and (K, A) in (U, A, τ). By hypothesis there exists a soft continuous map f : (U, A, τ) ⟶ [0, 1] such that f (F, A) = {0}, f (K, A) = {1}. Let a, b$\tilde{\in}$ R be arbitrary such that a ⩽ b. Write G = [0, a), H = (b, 1]. Then G and H are disjoint soft open sets in R. Soft continuity of f ⟹ f^{-1}(G, A) and f^{-1}(H, A) are soft open in (U, A, τ). By assumption f (F, A)={0}⟹f^{-1}{0} = f^{-1}f (F, A)$\tilde{\supseteq}$(F, A) ⟹ f^{-1}{0}$\tilde{\supseteq}$(F, A)⟹ (F, A)$\tilde{\subseteq}$ f^{-1}{0}.Similarly(K, A) $\tilde{\subseteq}$ f^{-1}{1}. Evidently {0}$\tilde{\subseteq}$ [0, a) ⟹ f^{-1}({0})$\tilde{\subseteq}$$f^{-1}$[0, a) ⟹ (F, A) $\tilde{\subseteq}$ f^{-1}({0}) $\tilde{\subseteq}$$f^{-1}$[0, a) ⟹ (F, A)$\tilde{\subseteq}$$f^{-1}$[0, a) ⟹ (F, A) $\tilde{\subseteq}$ f^{-1}(G) .Similarly(G,A) $\tilde{\subseteq}$ f^{-1}(H). f^{-1} (G) $\tilde{\cap}$ f^{-1}(H) = f^{-1}(G$\tilde{\cap}$H) = f^{-1}(Ø) = (Ø, A).

IV. CONCLUSION

Urysohn's lemma in soft topological spaces gives a considerable originality in its proof. It could be used in proving a number of important theorems in soft topology - namely Urysohn's metrization theorem, Tietze Extension theorem, imbedding theorem for manifolds etc.

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