Kerr-Newmann Black Hole Thermodynamics, Quantum Geometry and Information Theory

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There have been a lot of investigations going on in the field of black holes in AdS spacetime. Most of the works deal with the standard thermodynamic space where the role of pressure has never been explicit. In this paper, pressure term is incorporated and the fundamental equation of the Kerr-Newman black hole in AdS spacetime is accordingly calculated. Then using a thought experiment explained in this paper, the fundamental principles of general relativity and quantum mechanics are contrasted. This demands a new perspective to look at quantum mechanics and there by applying these informative geometric approaches applied to black holes, to the Hilbert space helps carry out the analysis in the quantum domain showing a direct analogy between the structure of general relativity and the Hilbert space. This is done by defining a second rank tensor which is analogous to rank two magnetic tensor potential, from where one gets the metric in Hilbert space and Berry's curvature. Further defining a new kind of derivative (in terms of density matrix) helps include time in the theory as well and takes care of the curvatures involved in quantum systems. The paper then continues to extend the study of information theory where it is shown that Neumann entropy is a super set of Shannon entropy and hence knowing density matrix of a quantum system is sufficient enough to have all of the information content of that quantum system as well as its classical limit. These studies show the fundamental importance of density matrices which control both the information content and the geometry of the system in question.

I. INTRODUCTION

The recent work by Dolan [2] has shown that there are non trivial definitions of pressure and volume, where the pressure is actually connected to the cosmological constant Λ. These definitions led to a search for the dependence of the internal energy U of black hole in AdS spacetime on pressure and this could be done only in an extended thermodynamic phase space where the result obtained is:

$$dU = TdS + \mathbf{\Omega}.\mathbf{dJ} + \Phi dQ - PdV \tag{1}$$

where T is the temperature, S is the entropy, $\mathbf{\Omega}$ is the angular velocity, $\mathbf{J}$ is the angular momentum, Φ is the potential, Q is the charge, P is the pressure and V is the volume of the black hole. These analyses were carried out further and led to a search for a general thermodynamic description of KN-black holes in AdS spacetime in extended phase space. Then carrying out the analogy between the general relativistic case [1] and non-relativistic quantum mechanics [13] using the concept of Ruppeiner geometry [3] as well as contrasting their corresponding fundamental principles in section IV, one gets a motivation to study quantum geometry [4] which helps to determine all the geometrical aspects of Hilbert space and also a method to analyze the points of quantum phase transitions. There comes an analogy with magnetic vector potential and a concept of magnetic tensor potential of rank two is defined in this context whose real part gives the metric and the imaginary part gives the Berry's curvature ,i.e., it is sufficient to determine all geometric properties of Hilbert space. Using the motivation that density matrices are used to arrive at all these, the paper proceeds to the information theory and shows Shannon entropy as a subset of Neumann entropy and hence, the information coming from the Neumann entropy and geometrical properties, being determined using density matrices, can be unified in a single framework of information geometry.

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II. KERR-NEWMANN BLACK HOLE IN ANTI-DE SITTER SPACETIME

The definition of ADM mass in AdS spacetime takes a non-trivial definition in terms of the enthalpy of the system there in question. The following result is obtained for the case of AdS spacetime:

$$dM = dH = TdS + \mathbf{\Omega}.\mathbf{dJ} + \Phi dQ + VdP \tag{2}$$

where H is the enthalpy and M is the ADM mass of the black hole [6]. Using this relation and the laws of thermodynamics, the fundamental equation of KN-black hole in the extended phase space is obtained as follows:

$$dS = \frac{1}{T}[\frac{\partial M}{\partial T}dT + (\frac{\partial M}{\partial Q} - \Phi)dQ + (\frac{\partial M}{\partial P} - V)dP + (\nabla_{\mathbf{J}} - \mathbf{\Omega}).\mathbf{dJ}] \tag{3}$$

where

$$\nabla_{\mathbf{J}} = \frac{\partial}{\partial J_x}\hat{\mathbf{i}} + \frac{\partial}{\partial J_y}\hat{\mathbf{j}} + \frac{\partial}{\partial J_z}\hat{\mathbf{k}} \tag{4}$$

Using this, equations of state can be easily deduced for this black hole [10]. They are as follows:

$$\frac{1}{T}\frac{\partial M}{\partial T} = \frac{\partial S}{\partial T} \tag{5}$$

$$\frac{1}{T}[\frac{\partial M}{\partial Q} - \phi] = \frac{\partial S}{\partial Q} \tag{6}$$

$$\frac{1}{T}[\frac{\partial M}{\partial P} - V] = \frac{\partial S}{\partial P} \tag{7}$$

$$\frac{1}{T}[\nabla_{\mathbf{J}} M - \mathbf{\Omega}] = \nabla_{\mathbf{J}} S \tag{8}$$

Then applying the condition for maximum entropy under the constraint of constant energy, the method to be employed is that of Lagrange multipliers [9], giving the following conditions on mass in extremal conditions:

$$\frac{\partial M}{\partial T} = 0 \ and \ \frac{\partial^2 M}{\partial T^2} < 0 \tag{9}$$

Hence maximizing entropy resulted in maximizing the ADM mass. When converted to thermodynamic variables, it is equivalent to demanding heat capacities to be always positive which is indeed the case of KN-black holes in AdS spacetime. The other results obtained on the first partial derivatives of mass of the black hole are:

$$\frac{\partial M}{\partial Q} = \phi \tag{10}$$

$$\frac{\partial M}{\partial P} = V \tag{11}$$

$$\nabla_{\mathbf{J}} M = \mathbf{\Omega} \tag{12}$$

Similarly there are upper bounds placed on the second partial derivatives of the mass of the black hole. They are:

$$\frac{\partial^2 M}{\partial Q^2} < \frac{\partial \phi}{\partial Q} \tag{13}$$

$$\frac{\partial^2 M}{\partial P^2} < \frac{\partial V}{\partial P} \tag{14}$$

$$\nabla^2_{\mathbf{J}} M < \nabla_{\mathbf{J}} \mathbf{\Omega} \tag{15}$$

III. RUPPEINER/THERMODYNAMIC GEOMETRY

The pioneering work of Ruppeiner [3] led to a new field of interest where using the curvature of the thermodynamic phase space, the phenomena of critical points and phase transitions can be explained. Using these techniques, the covariant version of classical thermodynamic fluctuation theory is chalked out which is invariant under any general coordinate transformation. There, line element is calculated in various thermodynamic representations, for instance, the entropy representation, where in particular, the metric is given by:

$$g_{\alpha\beta} = \frac{-1}{k_B}\frac{\partial^2 s}{\partial a^\alpha \partial a^\beta} \tag{16}$$

where s is entropy per unit volume and a's are thermodynamic coordinates in that particular representation. Applying these tools to calculate the Riemannian thermodynamic curvature scalar R, using RGTC (Riemannian Geometry and Tensor Calculus) package in Mathematica, for ideal gas, multicomponent ideal gas, Ising model, van der Waals gas and ideal paramagnet, one concludes to the proposition that R is inversely proportional to free energy, hence, R α correlation volume, below which classical thermodynamic fluctuation theory fails. This in turn implies that the points where R blows up are actually the critical points of the system, where critical phenomena occur. Points where the sign of R changes shows a phase transition.

Extending these analyses to black hole Physics, thermodynamic metric of KN-black hole in AdS spacetime is calculated in the extended phase space and found the metric to be positive definite, which is expected because of the stability issue, and the diagonal term turned out to be the heat capacity of such a black hole in extremal conditions, which is indeed positive for KN-black hole in AdS spacetime, showing thermodynamic stability. Recognizing ADM mass of the black hole to be the enthalpy of that black hole thermodynamic system in AdS spacetime [6], extremal conditions imply maximizing entropy by conserving the energy of the black hole as it is thermally stable and not losing energy, conditions on the first and the second derivative of the ADM mass of KN-black hole in AdS spacetime is calculated. Further extremal conditions implied temperature to be zero, showing a plausible breakdown of the third law of black hole thermodynamics, which states that it is impossible to reduce the temperature of a black hole to zero by a finite number of processes. Calculating R also shows that there is an inherent thermodynamic curvature present in terms of Ruppeiner geometry and it is not a case of flat geometry unlike Reissner-Nordström black hole in (3+1)D spacetime. Points where the sign of R changes shows a phase transition.

IV. CONSISTENCY RELATION

The purpose of this section is to show a consistency relation between general relativity and non-relativistic quantum mechanics. General relativity has all of its information contained in a geometrical picture while quantum mechanics operates on uncertainty, probability and complimentarity.

Principle of equivalence [11] and the linearity principle of quantum mechanics [13] are of paramount importance for the general relativity and quantum mechanics respectively to hold true. What if these 2 principles are contrasted, then the result derived is the required consistency relation for these two. Consider two situations:

- a quantum measurement being done where gravitational potential energy is included, then the Schrödinger's equation becomes:

$$i\hbar\frac{\partial\psi_1}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi_1 - mgx\psi_1 \tag{17}$$

- for the same experiment done in a freely falling lift, the Schrödinger's equation becomes:

$$i\hbar\frac{\partial\psi_2}{\partial t'} = \frac{-\hbar^2}{2m}\nabla'^2\psi_2 \tag{18}$$

Notations used are ($\overrightarrow{x}$, t) for equation (17) and ($\overrightarrow{x'}$, t') for equation (18).
Since Schrödinger's equation is being used, so it is in the non-relativistic realm. Then the coordinates of equations (17) and (18) are related as:

$$\overrightarrow{x'} = \overrightarrow{x} + \frac{1}{2}gt^2 \tag{19}$$

$$t' = t \tag{20}$$

Now invoking the principle of equivalence and since Physics should not change, hence the following condition is demanded: $|\psi_1|^2 = |\psi_2|^2$. This leads to the consistency relation:

$$\psi_2 = e^{(i/\hbar)(-\frac{mg^2t^3}{6}+mgxt)}\psi_1 \tag{21}$$

where non-linearity in time t is noticed. This shows that after some time gets elapsed, the non-linear term in time t will dominate hence breaking the stability of the linear superposition principle. The time scale where it happens is of the order

$$\Delta t \approx \frac{\hbar}{\Delta E} \tag{22}$$

where ΔE is the energy difference between the the states of the same system at two different times t_{final} and $t_{initial}$. Hence we see that the present state of quantum mechanics fails at this time scale when the geometrical picture of gravitation is taken into account. The correction to the linear quantum mechanics to take into account the non-linear behavior should come from a geometrical picture. Then the present state of quantum mechanics can be shown as a linear approximation of that non-linear theory. In the next section of this paper, there is an attempt to exploit the geometry of Hilbert space and deduce some of the properties, which is shown there.

V. QUANTUM GEOMETRY

Motivation for this section is a belief that from a geometrical point of view, one can explain the various phenomena of quantum origin [4], [5]. Inspired from the previous sections, let a new derivative be defined as:

$$|\nabla_\mu\psi> \equiv |\frac{\partial\psi}{\partial\lambda^\mu}> \mp i <\psi|\frac{\partial\psi}{\partial\lambda^\mu}> |\psi> \tag{23}$$

$$\Rightarrow |\nabla_\mu\psi> = |\frac{\partial\psi}{\partial\lambda^\mu}> \mp i\hat{\rho}|\frac{\partial\psi}{\partial\lambda^\mu}> \tag{24}$$

where $\hat{\rho} = |\psi><\psi|$ gives the density matrix.

For the sake of convenience, let it be called *proper derivative*. Here $\lambda = \{\lambda^\mu\}$ shows a set of parameters which can include time as well. This is in the parameter space. There is an ambiguity in sign as both the signs will give the same result. Point to be noted here is that Berry's connection (which is an analogue to the affine connection in general relativity but not exactly the same as will be seen later) is explicitly used to define it and there is no need to talk about it separately in the context of information geometry. Here Berry's connection is gauge dependent.

Then under this definition of proper derivative, if a quantity $A_{\mu\nu}$ is defined such that:

$$A_{\mu\nu} \equiv Re[|\nabla_\nu\psi><\nabla_\mu\psi|] + iIm[-2*|\nabla_\nu\psi><\nabla_\mu\psi|] \tag{25}$$

For the sake of brevity, say $A_{\mu\nu} = m + in$, then here m is symmetric in indices μ and ν and gives the metric $g_{\mu\nu}$ which in turn decides all the geometric properties in any parameter space, analogous to general relativity. The imaginary term n is anti-symmetric in indices μ and ν and gives the Berry's curvature which comes naturally as an outcome of defining the proper derivative in the above mentioned way. This again stresses the fact that there is no need to talk about Berry's curvature separately as this model unifies the approach. Here Berry's curvature is gauge independent.

First focus on the real part m and then switch to the imaginary part n. This reduces to the standard result of Provost and Vallee [4] for the definition of metric [15] but here time can also included and is generalized to arbitrary number of dimensions in the parameter space. To stress the Physics behind the real part m, use the perturbation technique to get:

$$|\psi_0(\lambda + d\lambda) > \sim |\psi_0(\lambda) > + \tag{26}$$

$$\sum_{n \neq = 0} \frac{|\psi_n(\lambda) >< \psi_n(\lambda)|H(\lambda + d\lambda) - H(\lambda)|\psi_0(\lambda) >}{E_0 - E_n}$$

This gives the definition of metric as follows, as given in Zanardi [7]:

$$g_{\mu\nu} = Re[\frac{\sum_{n \neq 0} < \psi_0(\lambda)|\partial_\mu H|\psi_n(\lambda) >< \psi_n(\lambda)|\partial_\nu H|\psi_0(\lambda) >}{|E_n(\lambda) - E_0(\lambda)|^2}] \tag{27}$$

where perturbation theory was used. The following relation of the line element in Hilbert space is calculated using the formula of quantum metric:

$$\hbar ds = 2\Delta E dt \tag{28}$$

where ds is the line element, dt is the time interval and ΔE is the energy uncertainty (same as that appearing in energy-time uncertainty relation). This can be obtained in an alternative manner as well, which is done in [5], using the method of perturbative expansion and truncating up to 1st order terms. This relation shows how the fuzziness enters into the dynamics of the quantum system in its geometry.

The real part of $A_{\mu\nu}$,i.e., m gives the same and this highlights the Physics behind it. To make it in the proper form, replace $\partial_\mu \to \nabla_\mu$ and $\partial_\nu \to \nabla_\nu$ and by doing so, one gets the proper form. Then using m, one can continue to find other properties like Riemannian curvature tensor, curvature scalar and other geometric properties. The general form of affine connection (assuming $g_{\mu\nu}$ to be symmetric in μ and ν) is:

$$\Gamma_{\alpha\mu\nu} = \frac{g_{\alpha\mu,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha}}{2} \tag{29}$$

$$\Rightarrow \Gamma_{\alpha\mu\nu} =< \frac{\partial^2\psi}{\partial\lambda^\mu\partial\lambda^\nu}|\frac{\partial\psi}{\partial\lambda^\alpha} > + < \frac{\partial^2\psi}{\partial\lambda^\mu\partial\lambda^\nu}|\psi >< \frac{\partial\psi}{\partial\lambda^\alpha}|\psi > \tag{30}$$

When Berry's connection = 0, then proper derivative = normal partial derivative, but this not sufficient to make affine connection go to zero, however this has other important significance as well which is highlighted below.

Now turning the attention to the imaginary part n which shows the Berry's curvature, start with the definition of it in terms of the above defined proper derivative. Let Berry's connection be denoted as C_μ which has the standard definition as:

$$C_\mu = i < \psi|\frac{\partial\psi}{\partial\lambda^\mu} > \tag{31}$$

This implies Berry's curvature $\Omega_{\mu\nu}$ to be:

$$\Omega_{\mu\nu} = C_{\nu;\mu} - C_{\mu;\nu} \tag{32}$$

where ; denotes proper derivative. Here either on plugging equation (23) or equation (24), one gets:

$$\Omega_{\mu\nu} = C_{\nu,\mu} - C_{\mu,\nu} \tag{33}$$

where , denotes simple partial derivative. This is expected from the knowledge of differential geometry [11], showing internal consistency of the theory developed here. To again highlight the Physics behind it, one can use the perturbation theory to get the form of Berry's curvature as follows:

$$\Omega_{\mu\nu} = i \sum_{n' \neq n} \frac{< \psi_n|\partial_\mu H|\psi_{n'} >< \psi_{n'}|\partial_\nu H|\psi_n > -(\nu \leftrightarrow \mu)}{|E_n - E_{n'}|^2} \tag{34}$$

Again to get the proper form, replace $\partial_\mu \to \nabla_\mu$ and $\partial_\nu \to \nabla_\nu$. When Berry's connection $C_\mu = 0$ throughout, one gets:

$$Im[|\nabla_\nu\psi >< \nabla_\mu\psi|] = 0 \tag{35}$$

and

$$\Gamma_{\alpha\mu\nu} =< \frac{\partial^2\psi}{\partial\lambda^\mu\partial\lambda^\nu}|\frac{\partial\psi}{\partial\lambda^\alpha} > \tag{36}$$

Hence Berry's curvature turns out to be zero.

As a final test and check for internal consistency, let's calculate the commutation relation of the above defined proper derivatives. It comes out:

$$[\nabla_\mu, \nabla_\nu]|\psi >= \pm(\pm < \partial_\mu\psi|\psi >< \psi|\partial_\nu\psi > \tag{37}$$

$$\mp < \partial_\nu\psi|\psi >< \psi|\partial_\mu\psi >)$$

$$\pm i(\pm < \partial_\nu\psi|\psi >< \psi|\partial_\mu\psi > \mp < \partial_\mu\psi|\psi >< \psi|\partial_\nu\psi >) \neq 0$$

The expression is not that important as not being equal to zero because one expects them to non-commute and hence on going on a closed curve, when one reaches the starting point again (end point = starting point), that is not same as the original starting point when the journey started from there. This fact is taken care in the theory developed here starting from the definition of proper derivative and its consequences in the form of Berry's connection and Berry's curvature. Hence this cross check is confirmed.

Hence it can be concluded that $A_{\mu\nu}$ gives all the geometrical properties in a single framework including metric, Berry's connection, phase and curvature. Moreover, near quantum phase transitions, $A_{\mu\nu}$ (both the real and the imaginary part) blows up which is also evident from the perturbation theory. Also, the curl of $A_{\mu\nu}$ over a closed loop turns out to be zero, so it resembles in some sense the magnetic flux which is also zero over a closed surface (Gauss's law for magnetism). This creates an analogy between magnetic vector potential and $A_{\mu\nu}$ acting like a second rank magnetic tensor potential.

Further motivation comes from the fact that the expression given by $\hat{\rho} = |\psi >< \psi|$ gives the density matrix, which further gives the Neumann entropy as $S = -Tr[\rho log_2 \rho]$ (log_2 is just a convention used by information theorists, physicists using the convention $log_e = ln$). The only difference between this expression of Neumann entropy and that of the expression of $A_{\mu\nu}$ is that of a proper derivative. Hence this provides a motivation that if Neumann entropy can be shown as a basis for Shannon entropy (Shannon entropy being the subset of Neumann entropy), then information coming from the Neumann entropy and geometrical properties coming from the $A_{\mu\nu}$ can be unified in a single framework of information geometry. This is the motivation for the next section.

VI. INFORMATION THEORY

Consider a two state system with variables A and B with eigenvalues a and b whose occurrence has probabilities p(a) and p(b) respectively.

A. Conditional Entropy

Let it be defined as:

$$S(A|B) \equiv -Tr[\rho_{AB} log_2 \rho_{A|B}] \tag{38}$$

where $S(A|B)$ is the entropy corresponding to state A, once B is known completely and $\rho_{A|B}$ is defined as:

$$\rho_{A|B} \equiv \frac{\rho_{AB}}{I_A \otimes \rho_B} \tag{39}$$

where I_A is the unit matrix in the Hilbert space of A. On plugging equations (39) in (38), one gets the expected result:

$$S(A|B) = S(AB) - S(B) \tag{40}$$

which confirms the definition [12]. This definition generalizes to the Shannon entropy when density matrices are diagonal.

Here $\rho_{A|B}$ is a positive Hermitian operator whose eigenvalues can cross 1 which means that actually its not a density matrix. When it crosses one, $S(A|B)$ becomes negative. This is a necessary and sufficient condition for states A and B to be entangled.

Another way to look at it is as follows. On knowing B, there can only be three possibilities here:

1 . nothing is known about A (i.e., states are orthogonal), then

$$p(a|b) = \frac{p(ab)}{p(b)} = p(a) \tag{41}$$

and

$$S(A|B) \geq 0 \tag{42}$$

2 . something but not all is known about A, then

$$S(A|B) \geq 0 \tag{43}$$

3 . all is known about A (entangled states), then

$$S(A|B) < 0 \tag{44}$$

This is the necessary and sufficient condition for states to be entangled.

First two cases are the classical cases, covered by Shannon entropy's method of information theory. Neumann entropy covers the third case also (which is a purely quantum mechanical one), making Shannon entropy a subset of it.

NOTE:

1 . For entangled states, $S(A|B) < 0$, which means that either $S(A) > S(AB)$ or $S(B) > S(AB)$ making Neumann entropy a non-monotonically increasing function, which is a purely quantum mechanical effect.

2 . $S(A|B) \geq 0$ is the necessary and sufficient condition for the separability of states in Hilbert space

B. Mutual/Correlation Entropy

Let it be defined as:

$$S(A:B) \equiv -Tr[\rho_{AB} log_2 \rho_{A:B}] \tag{45}$$

where $S(A:B)$ is the entropy corresponding to state A, once something is known about B showing a correlation between these two states and $\rho_{A:B}$ is defined as:

$$\rho_{A:B} \equiv \frac{\rho_A \otimes \rho_B}{\rho_{AB}} \tag{46}$$

Here there are three cross-checks:

1 . On plugging equation (46) in equation (45), one gets the expected result:

$$S(A:B) = S(A) + S(B) - S(AB) \tag{47}$$

which confirms the definition [12].

2 . $S(A:B)$ is symmetric in A and B which should be the case as that holds for in case of classical case of Shannon entropy $H(A:B)$.

3 . $S(A:B) \geq 0$ always just like the classical case which matches the physical intuition that entropy of A can only be reduced or can remain constant (and never increased) by having some knowledge about B.

This definition again generalizes to the Shannon entropy when density matrices are diagonal.

As seen earlier, either $S(A) > S(AB)$ or $S(B) > S(AB)$ in case of entangled states, the classical identity $H(A:B) \leq min[H(A), H(B)]$ is broken and there arises a possibility:

$$S(A:B) > min[S(A), S(B)] \tag{48}$$

which is the zone of entangled states and shows a hyper correlation between the 2 states. There exists an upper bound to this which holds for maximally entangled states (like EPR pairs), which is as follows:

$$2\, min[S(A), S(B)] \geq S(A:B) > min[S(A), S(B)] \tag{49}$$

This is exactly the same as requiring $S(A|B) < 0$ for entangled states but is more useful because practically correlation entropy is used more than conditional entropy. The reason being that practically it is always much more difficult to know everything about B (which is the case of conditional entropy) rather than having some knowledge about B (which is the case of correlation entropy), obviously.

This again stresses that Shannon entropy is a subset of Neumann entropy as Neumann entropy covers the information theoretic aspects of both the classical as well as the purely quantum mechanical phenomena.

VII. CONCLUSION

As seen in this paper, there is a rich field which can be explored in quantum vector space in terms of quantum information and curvature properties. These techniques can be a starting point to give a firm theoretical grounding to some of the postulates of quantum mechanics. As an example, there is a possibility that if bosons and fermions can be explained in terms of positive and negative curvatures respectively as well, then their behaviors will have a geometrical reasoning for their properties as well which can be looked at. Further by the consistency relation, it can be concluded that quantum mechanics needs corrections coming from a geometrical point of view. THis paper establishes the importance of density matrix beyond doubt, which contains the information content of the system in question and with the help of $A_{\mu\nu}$, it tells about the geometrical properties of the system. Using density matrix, Neumann entropy can be computed which is a super set of Shannon entropy. Hence for the classical systems, Shannon entropy can be used to get the geometrical properties using (16). This is valid for classical domain only because an important point to note is that the concept of Ruppeiner geometry is applicable only in the classical domain as can be seen in [14]. There it has been shown that the conjecture proposed by Ruppeiner ([3]) that scalar curvature R due to the metric given by (16) is related to correlation length as $|R| \ \alpha \ \xi^d$ (d is the dimension of the system), does not hold in the quantum domain. As a far fetched and remote goal, the problem statement of quantum measurement can be formulated in a much nicer, explicit and clear way. This paper can be extended to analyze in terms of quantum scalar fields where it is shown in [8] that the ground state density matrix for a massless free quantum field is traced over the degrees of freedom living inside an imaginary sphere and the resulting entropy of this configuration is shown to be proportional to the area (and not the volume) of that imaginary sphere, showing a deep analogy with black hole Physics.

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$$g_{\mu\nu} = |\frac{\partial\psi}{\partial\lambda^{\nu}} >< \frac{\partial\psi}{\partial\lambda^{\mu}}| + <\psi|\frac{\partial\psi}{\partial\lambda^{\nu}} >< \frac{\partial\psi}{\partial\lambda^{\mu}}|\psi> \tag{50}$$