

Image Compression and Denoising Effects using Wavelets

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Abstract: In the present work we analyze the performance of Bi-orthogonal wavelet filters for image compression and denoising on variety of test images. The test images are of different size and resolution. The compression performance is measured, objectively peak signal to noise ratio and subjectively visual quality of image. The paper also deals with the use of wavelet transform for signal and image de-noising employing a selected method of thresholding of appropriate decomposition coefficients. The proposed technique is based upon the analysis of wavelet transform and it includes description of global modification of its values.

Keyword: Denoising, DWT Transform, PSNR ratio, Wavelets

I. INTRODUCTION

An enormous amount of data is produced when 2-dimensional light intensity function is sampled and quantized to create a digital image. In fact the amount of storage is so great that it creates problem in practical storage, processing and communication requirement. Uncompressed multimedia (graphics, audio and video) data requires considerable storage capacity and transmission bandwidth. Despite rapid progress in mass-storage density, processor speeds, and digital communication system performance, demand for data storage capacity and data-transmission bandwidth continues to outstrip the capabilities of available technologies. The recent growth of data intensive multimedia-based web applications have not sustained the need for more efficient ways to encode signals and images but have made compression of such signals to storage and communication technology[1].

Wavelet transform is one of the promising methods of image compression[2]. It has received significant attention recently due to their suitability for a number of image processing tasks including image compression. The basic measure of the performance of a compression algorithm is the compression ratio and peak signal to noise ratio, which is defined by the ratio between original data size and compressed data size[3]. According to this analysis, we show the selection of the optimal wavelet for image compression taking into account Peak Signal to Noise Ratio (PSNR) as objective and visual quality of image as subjective quality measures.

II. WAVELET TRANSFORM

Wavelet transform (WT) represents an image as a sum of wavelet functions (wavelets) with different locations and scales[4]. Any decomposition of an image into wavelets involves a pair of waveforms: one to represent the high frequencies corresponding to the detailed parts of an image (wavelet function) and one for the low frequencies or smooth parts of an image (scaling function).

A. Discrete Wavelet Transform

One of the big discoveries for wavelet analysis was that perfect reconstruction filter banks could be formed using the coefficient sequences k and l (Fig. 1). The input sequence x is convolved with high-pass (HPF) and low-pass (LPF) filters and each result is down sampled by two, yielding the transform signals w and s . The signal is reconstructed through up sampling and convolution with high and low synthesis filters h and g . For properly designed filters, the signal is reconstructed exactly ($y=x$) [5].

The choice of filter not only determines whether perfect reconstruction is possible, it also determines the shape of wavelets we use to perform the analysis. By cascading the analysis filterbank with itself a number of times, a digital signal decomposition with dyadic frequency scaling known as DWT can be formed [6]. The mathematical manipulation that effects synthesis is called inverse DWT. An efficient way to implement this scheme using filters was developed by Mallat. The new twist that wavelets bring to filter banks is connection between multiresolution analysis (that, in principle, can be performed on the original, continuous signal) and digital signal processing performed on discrete, sampled signals. DWT for an image as a 2-D signal can be derived from 1-D DWT. The easiest way for obtaining scaling and wavelet function for two dimensions is by multiplying two 1-D functions. The scaling function for 2-D DWT can be obtained by multiplying two 1-D scaling functions. Wavelet functions for 2-D DWT can be obtained by multiplying two wavelet functions or wavelet and scaling function for 1-D analysis. For the 2-D case, there exist three wavelet functions that scan details in horizontal, vertical, and diagonal directions [7].

This may be represented as a four-channel perfect reconstruction filter bank as shown in Fig. 1/. Now, each filter is 2-D with the subscript indicating the type of filter (HPF or LPF) for separable horizontal and vertical components. The resulting four transform components consist of all possible combinations of high- and low-pass filtering in the two directions [8]. By using these filters in one stage, an image can be decomposed into four bands. There are three types of detail images for each

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resolution: horizontal (HL), vertical(LH), and diagonal (HH). The operations can be repeated on the low-low band using the second stage of identical filterbank[9]. Thus, a typical 2-D DWT, used in image compression, will generate the hierarchical pyramidal structure shown in Fig. 2. Here, we adopt the term “number of decompositions” to describe the number of 2-D filter stages used in image decomposition. Wavelet multiresolution and direction selective decomposition of images is matched to an HVS. In the spatial domain, the image can be considered as a composition of information on a number of different scales. A wavelet transform measures gray-level image variations at different scales. In the frequency domain, the contrast sensitivity function of the HVS depends on frequency and orientation of the details.

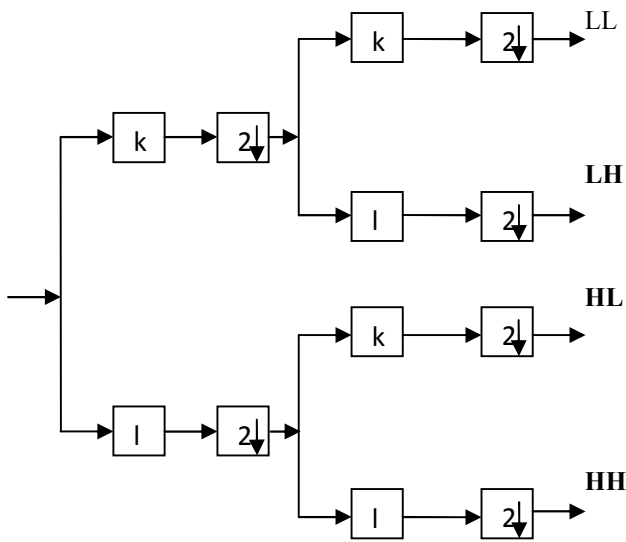


Fig 1. Single stage 2-D wavelet expansion

LL	HL3	HL2	HL1
LH3	HH3		
LH2		HH2	
LH1		HH1	

Fig 2. Three level wavelet expansion

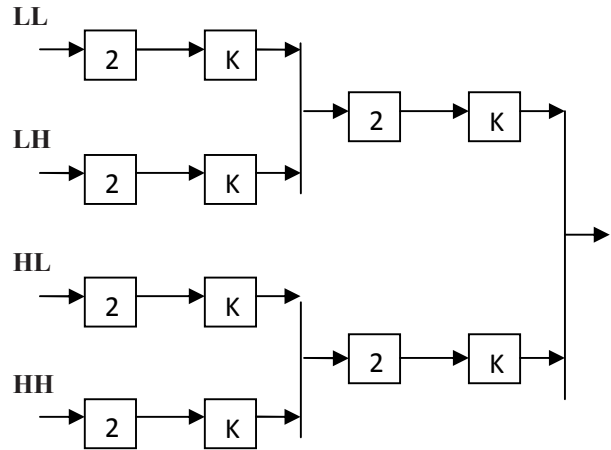


Fig 3. Single stage 2-D wavelet reconstruction

III. WAVELET THRESHOLDING

Suppose $x = \{x_{ij}, i=1,2,\dots,M \text{ and } j=1,2,\dots,N\}$ is an image of $M \times N$ pixels, which is corrupted by independent and identically distributed (i.i.d) zero mean, white Gaussian noise n_{ij} with standard deviation σ_n . The noise signal can be denoted as $n_{ij} \sim N(0, \sigma_n^2)$. This noise may corrupt the signal in a transmission channel. The observed, noise contaminated, image is $y = \{y_{ij}, i=1,2,\dots,M \text{ and } j=1,2,\dots,N\}$. Therefore, the noised image can be expressed as:

$$y_{ij} = x_{ij} + n_{ij}$$

The object of a de-noising process is to estimate image x from the noised image y , so that the Mean Square Error (MSE) to be minimum. Let W and W^{-1} denote the two dimensional DWT and its inverse respectively. Then, the original signal, its noised version and the noise have a matrix form in the transform domain that includes the sub band coefficients.

$$X = Wx, Y = Wy, V = Wn$$

Fig.4. shows the two level DWT of a 2-D signal, which consists of the sub bands LL_2 (low frequency or approximation coefficients), HL_2 (horizontal details), LH_2 (vertical details), HH_2 (diagonal details) and the first level details HL_1, LH_1, HH_1 .

LL	HL3	HL2	HL1
LH3	HH3		
LH2		HH2	
LH1		HH1	

Fig. 4. Pyramidal structure of a wavelet decomposition

In the spatial domain, becomes in the transform domain as follows:

$$Y = X + V$$

where X, Y and V are the transform domains of the original image, its noised version and the noise respectively. The orthogonal property of the transform insures that the noise in the transform domain is also of Gaussian nature. The denoising algorithms, which are based on thresholding, suggest that each coefficient of every detail subband is compared to a threshold level and is either retained or killed if its magnitude is greater or less respectively. The approximation coefficients are not submitted in this process, since on one hand they carry the most important information about the image and on the other hand the noise mostly affects the high frequency subbands. The type of the threshold is either hard or soft. Fig. indicates the two types of thresholding, which can be expressed analytically as follows.

$$\text{Hard threshold: } \begin{cases} y = x & \text{if } |x| > T \\ y = 0 & \text{if } |x| < T \end{cases}$$

$$\text{Soft threshold: } y = \text{sign}(x) (|x| - T)$$

Where x is the input signal, y is the signal after threshold and T is the threshold level.

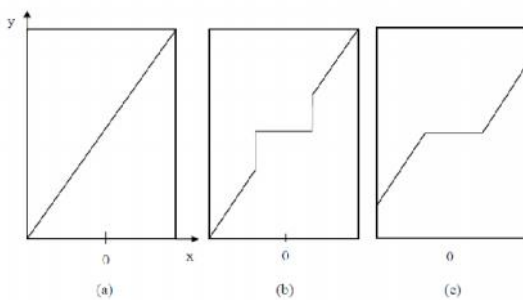


Fig 5. Threshold types: (a) Original signal; (b) Hard; (c) Soft

The hard type does not affect the coefficients that are greater than the threshold level, whereas the soft type causes shrinkage to these coefficients. In the present work, both types of threshold are evaluated but hard thresholding may create abrupt artifacts because of its discontinuous nature. The reconstructed image is a de-noised estimate of x, which is produced by the inverse DWT.

$$\hat{x} = W^{-1} \hat{Y}$$

Where \hat{Y} consists of the thresholded subbands of the noised image.

The threshold level is estimated by various methods called thresholding criteria, which are based on the minimization of the averaged squared error.

$$\arg \min \left[\frac{1}{N} \sum_i (\hat{Y}_i - X_i)^2 \right]$$

where X_i and \hat{Y}_i all the detail subbands coefficients of the original image and the noised image after thresholding respectively.

IV. QUALITY MEASURES

The performances of image compression techniques are mainly analyzed on the basis of two measures: Compression Ratio (CR) and Peak Signal to noise ratio (PSNR). The compression ratio is defined as ratio of the size of original data set to the size of the compressed data set

$$\text{Compression ratio} = \frac{A}{B} * 100$$

Where A = Number of Bytes in the original data set

B = Number of Bytes in the Compressed data set

PSNR provides a measurement of the amount of distortion in a signal, with a higher value indicating less distortion. For n-bits per pixel image, PSNR is defined as:

$$\text{PSNR} = 20 \log_{10} \frac{2^R - 1}{\text{RMSE}} \text{ db}$$

Where, RMSE is the root mean square difference between two images. The Mean Square Error (MSE) is defined as follows:

$$\text{MSE} = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} [y(m, n) - x(m, n)]^2$$

where x(m,n), y(m,n) are respectively the original and recovered pixel values at the mth row and nth column for MxN size image. The PSNR is given in decibel units (Db), which measures the ratio of the peak signal and the error signal (difference between two images). The PSNR value provides the quality objectively. While, visual quality of image is considered as subjective quality measures.

V. RESULTS

We analyze and Biorthogonal wavelet families for image compression and denoising and also compare their results. We used two types of test images with different resolution and different size

Also, it is found that the wavelet function BIOR 2.6 gives better compression performance for large size images in terms of PSNR values. While wavelet functions BIOR 2.4 shows the competitive compression performance for the small size images. The analysis and comparison of the results show that not only in the BIOR family, the wavelet function BIOR 2.6 gives the better compression performance (in terms of PSNR) in all the wavelet families considered in our experiment. This shows that objective as well as subjective quality of the compressed image is better for Biorthogonal wavelet family. Reason behind this performance is that Biorthogonal wavelets can use filters with similar or dissimilar order for decomposition (Nd) and reconstruction (Nr). Therefore Biorthogonal wavelet is parameterized by two numbers and filter length is {max(2Nd, 2Nr) + 2} [8]. Also these are Symmetric and Symmetry provides linear phase and minimize border artifacts.

In study if decomposition level is increased the compression performance improves but the quality of image deteriorates. Further, it is also observed that the BIOR wavelet families take much more computational time. Also it is found that as

the filter order increases in a given wavelet family, the compression performance increases, but the visual quality of compressed image becomes poorer. The higher order of filters involves the longer filters, which involves more blurring in the compressed image.

Table.1 Wavelet Family: Biorthogonal (compression)

PSNR in db					
Image	BIOR 1.3	BIOR 1.5	BIOR 2.2	BIR 2.4	BIOR 2.6
Came raman	21.74	21.48	22.90	22.92	23.00

Table2. Wavelet Family: Biorthogonal (denoising)

PSNR in db					
Image	BIO R1.3	BIOR 1.5	BIOR 2.2	BIR 2.4	BIOR 2.6
camer aman	61.7 4	61.61	61.86	22.92	61.94
Rice	62.8 5	62.79	62.14 7	62.147 5	62.09

VI. CONCLUSION

This study presents an analysis and comparison of the wavelet families for image compression considering PSNR and visual quality of image as quality measure. The effects of Biorthogonal wavelet families on test images are examined. We analyzed the results for a wide range of wavelet families and found that the wavelet BIOR2.6 provides best compression performance for all variety of images almost at all the compression ratios among all the families considered. The computational time required for the Biorthogonal wavelet family is more in comparison to other wavelet families.

VII. REFERENCES

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