Performance of Cramer Rao Bound on Jitter Variance Estimation for Blind Channel

D.Madhavi and Dr.M.Ramesh Patnaik

Abstract: In this paper, we compare the Cramer-Rao bounds (CRB) under the linear and norm constraints, for symbol and channel response estimation. We consider the problem of estimating parameters of an irregular sampling process defined as a uniform sampling process in which the deviations from the nominal sampling times constitute a random independent identically distributed (IID) process (jitter). For multi channel FIR systems (SIMO), it has been shown that the norm-constrained CRB is smaller than the linear-constrained CRB. And also the estimator's variance is compared to the linear and norm constraints. Computer simulations verify the effectiveness of the proposed estimation method for small jitter.

Keywords: jitter, cramer rao bounds, vector, SNR,

1. INTRODUCTION

In modern communication, SNR estimation ply more important role. First in this process, analog signal converts into discrete signal and this sampling process consist of a systematic random timing error (jitter). A related problem to jitter detection is the problem of estimation of jitter parameters. An efficient estimation procedure can be applied, for example, in classification of jammers and identification of friendly sources in ECM systems. Unlike a jitter detection, which under the conditions stated above, can be done only in the bi-spectral domain. It is provided that the sampling rate is no smaller than Nyquist rate, estimation of jitter parameters can be done also the spectral domain. The question as to which domain is preferable for carrying out the estimation is considered and it is shown using Cramer-Rao bounds. The answer depends on the skewness of the continuous process. If the skewness is larger than one, then its noise variance decreases. In this work we present an algorithm for estimation of the jitter variance using the bi-spectrum. This procedure can be adapted to the estimation of other jitter parameters, as long as they affect by the characteristic function of the jitter. The advantage of estimating jitter variance is that the jitter detection and estimation problems can be solved simultaneously. Specifically, the sampling is assumed to be uniform with additive independent identically distributed (IID) jitter.

D.Madhavi is working as Associate Professor in Dept. of ECE, GITAM University and Dr.M.Ramesh Patnaik is working as assistant Professor, Dept. of Instrument technology, A.U College of Engineering, Andhra university. Emails: dinokingkong945@gmail.com, ramesh_patnaik@yahoo.com

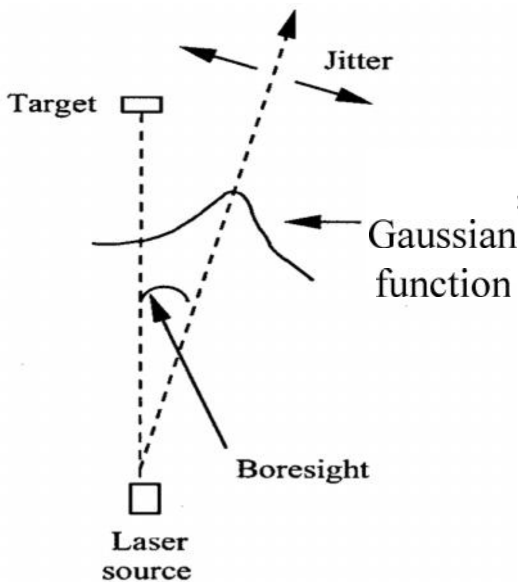


Fig1: jitter

A key question here: what is the ultimate performance limit that any estimation technique can achieve with respect to jitter and boresight estimation using the return photon signal? In this paper, we derive the Cramer–Rao lower bound (CRLB) on this estimator variance. And also compare the performance of linear and norm constrains by using CRLB. With the help of Gaussian far-field irradiance profile, we consider *N* shots directed from a laser source at a target in the far field as shown in Fig. 1.

IID process is a discrete time random process, X(n),(or random sequence X(1), X(2) · · · · · ·) is called an IID process earn mx (n) of IID process X(n) is a constant IID is represented by (1)

$$R_X(n_1, n_2) = E[X(n_1)X(n_2)] \quad (1)$$

$$= \begin{cases} E[X(n_1)] \cdot E[X(n_2)], & \text{if } n_1 < n_2 \\ E[X^2(n_1)], & n_1 = n_2 \end{cases}$$

$$= \begin{cases} m^2 & , \text{if } n_1 < n_2 \\ m^2 + \sigma^2 & , n_1 = n_2 \end{cases}$$

where

$$\delta n_1, n_2 = \begin{Bmatrix} 1, & n_1 = n_2 \\ 0, & n_1 \neq n_2 \end{Bmatrix}$$
$$\delta n_1, n_2 = \delta(n_1 - n_2)$$
$$C_X(n_1, n_2) = R_x(n_1, n_2) - m_x(n_1).m_x(n_2)$$
$$C_X(n_1, n_2) = \sigma^2.\delta n_1, n_2$$

First let us consider **a** communication system consisting of ***M*** channels. The complex finite impulse response of the i[th] channel is denoted by *hi(O),. ..>h ,(L),* ***a*** = **1,2,** ..., ***hi*.** *L* is the maximum order of all *M* channels, in the sense that ***h;(L)*** # 0 for at least one ***I*** and *hj(0)* # 0 for at least one ***j .*** **All** channels are driven by the same complex input symbol sequence so..., ***S N + L - ~*** transmitted from a single user. If we let **yi(n),n** = ***O>*** ..., ***8.-*** 1, be the output samples at ***N*** different time instants, for the i[th] channel, the output samples of the i[th] channel can be written by (2) and its parameters written as (3), (4), (5) & (6). Jitter variance represented by (9)

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{s} + \mathbf{w}_i = \bar{\mathbf{S}}\mathbf{h}_i + \mathbf{w}_i, \quad i = 1, 2, ..., M \tag{2}$$

where $\mathbf{y}_i = [y_i(0), y_i(1), ..., y_i(N-1)]^T$,

$$\mathbf{H}_i = \begin{bmatrix} h_i(L) & \cdots & h_i(0) & & \\ & \ddots & & \ddots & \\ & & h_i(L) & \cdots & h_i(0) \end{bmatrix}_{N\times(N+L)} \tag{3}$$

$$\mathbf{s} = [s_0, ..., s_{N+L-1}]^T \tag{4}$$

$$\bar{\mathbf{S}} = \begin{bmatrix} s_L & \cdots & s_0 \\ s_{L+1} & \cdots & s_1 \\ \vdots & \ddots & \vdots \\ s_{N+L-1} & \cdots & s_{N-1} \end{bmatrix} \tag{5}$$

$$\mathbf{h}_i = [h_i(0), \cdots, h_i(L)]^T \tag{6}$$

Blind symbol/channel estimation is to estimate symbol vector **s** and channel response vector h =**[hr,** ...,hLIT, from the noisy measurement ***yi,i*** =1>2>... , ***AI.*** **As** is shown in [4, 10], one complex parameter in ***[sT,*** **hT]** is inherently non-identifiable. To obtain a unique solution, constraints have to been incorporated into symbol/channel estimation approaches. Two widely used constraints include (a) the linear constraint: the lo-th (complex) element of the solution to symbol (of channel response) vector is set to one, and (b) the norm constraint: the norm of the solution to symbol (or channel response) vector is set to one and the lo-th element of the concerned solution vector is restricted to be real

Problem Formulation

Let us assume the noise component W_i is independent, jitter variance ***ut*** and **w;,** ***i*** = 1,2, ..., *M,* are also independent of each other Denote the linear-constrained symbol vector by (7) and the norm-constrained symbol vector by (8)**.** For real symbols/channel response, linear and norm constraints were discussed in (3)-(5). In the context of complex symbols, the linear constraint functions are given by equation

$$\mathbf{f}_l = \begin{Bmatrix} \Re e(\gamma_{l_0}) - 1 \\ \Im m(\gamma_{l_0}) \end{Bmatrix} = \mathbf{0} \tag{7}$$

$$\mathbf{f}_n = \begin{Bmatrix} (\|\rho\|^2 - 1)/2 \\ \Im m(\rho_{l_0}) \end{Bmatrix} = \mathbf{0} \tag{8}$$

$$\frac{\partial \mathbf{f}_l}{\partial \theta_l^T} = [\mathbf{0}_{2\times 2M(L+1)}, \mathbf{G}_l^T]$$

$$\frac{\partial \mathbf{f}_n}{\partial \theta_n^T} = [\mathbf{0}_{2\times 2M(L+1)}, \mathbf{G}_n^T]$$

Where,

$$\mathbf{G}_l = [\mathbf{e}_{l_0}, \mathbf{e}_{N+L+l_0}]$$

$$\mathbf{G}_n = [\begin{bmatrix} \Re e(\rho) \\ \Im m(\bar{\rho}) \end{bmatrix}, \mathbf{e}_{N+L+l_0}],$$

$$\dot{\mathbf{S}} = \begin{bmatrix} \Re e(\mathbf{S}) & -\Im m(\mathbf{S}) \\ \Im m(\mathbf{S}) & \Re e(\mathbf{S}) \end{bmatrix}$$

$$\dot{\mathbf{H}} = \begin{bmatrix} \Re e(\mathbf{H}) & -\Im m(\mathbf{H}) \\ \Im m(\mathbf{H}) & \Re e(\mathbf{H}) \end{bmatrix}$$

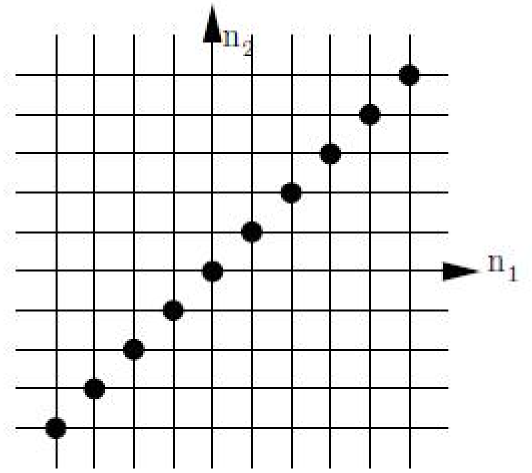
$$K = \begin{bmatrix} \sigma_\omega^2 & 0 & 0 \\ 0 & \sigma_\omega^2 & 0 \\ 0 & 0 & \sigma_\omega^2 \end{bmatrix} \tag{9}$$

Assume that mean IID of both noise and signals are zero with variances *σ2n* and *σ2s*, respectively, the covariance of the channel estimation error is approximated by the absolute CRB should be obtained from the most original equation (9). However, since the performance of the provided algorithm is only related (12), we would like to resort to the CRB after the pre-processing of our re-modulation. By using this CRB, we can also build up its relationship with the MSE that is derived in the previous subsection. The deterministic CRB for CPSOS will be considered. Here, where the observations are **z** = [**z***T*1 *, ...,* **z***T M*]*T*, and the parameters to be estimated are ***θ*** = [vec(**H**)*,* **d**1, ..., **d***M, σ2n*].

To calculate CRB, we need the joint probability density function (PDF) of **z**, denoted as *p*(**z**|***θ***). Since ***ηi*** is correlated with both ***ηi***−1 and ***ηi***+1, the covariance matrix of **z**, denoted as **R***z*, is an *MJ*(*N* +*L*)×*MJ*(*N* +*L*) Toeplitz matrix with the main diagonal elements 2, the (*JN* +1)th, −(*JN* +1)th2 diagonal elements −1, and all other elements are 0. We note that the inverse of such a huge Toeplitz matrix is mathematically represented as (10,11).

$$ACRB_{vec(H)} = I_K \otimes \left(\frac{\sigma_\omega^2 (\mathrm{K}^H)^\tau K^\tau}{2M\sigma_\omega^2} \right) \tag{10}$$

$$ACRB_{vec(H(1,k)} = \frac{\sigma_\omega^2 (\mathrm{K}^H)^\tau K^\tau}{2M\sigma_\omega^2} \tag{11}$$



$$r[n] = K \exp\left\{\frac{-[(x[n]+A)^2 + y^2[n]]}{2\omega^2}\right\}, \quad n = 1,2,...,N, \quad (12)$$

where K is a gain constant, ω is the noise standard deviation of the far-field irradiance pattern along any direction, $x[n]$ and y[n] are the angular coordinates for the n^{th} observation, and A is th boresight. The boresight represents the mean offset between the beam peak and the desired target (Fig. 1). The exact beam pointing is a random variable due to factors such as tracking errors, mechanical vibrations, and atmo where _j is the jitter standard deviation along any direction. In our subsequent discussions, we refer to _j simply as jitter. The target is assumed to be a point target in our model. The log pdf of the observation vector **r** = r[1], r[2] , . . . , r[N] for a given set of K, A, and _j is obtained by following equqtions.

$$\log p(r; A, \sigma_j) = 2N \log \Omega - \sum_{n=1}^{N} \log r[n] - 2N \log \sigma_j - \frac{1}{2\sigma_j^2}\left[A^2 N + 2\Omega^2 \sum_{n=1}^{N} \log(K/r[n])\right] \sum_{n=1}^{N} \log\left\{I_0\left[\frac{A}{\sigma_j^2}\sqrt{2\Omega^2 \log(K/r[n])}\right]\right\}$$

$$\frac{\partial \log p(r; A, \sigma_j)}{\partial A} = -\frac{AN}{\sigma_J^2} + \frac{1}{\sigma_J^2}\sum_{n=1}^{N}\frac{I_1(u[n])}{I_0(u[n])}\sqrt{z[n]}, \quad (13)$$

$$\frac{\partial \log p(r; A, \sigma_j)}{\partial \sigma_J} = -\frac{2N}{\sigma_J} + \frac{A^2 N}{\sigma_J^3} + \frac{1}{\sigma_J^3}\sum_{n=1}^{N} z[n] - \frac{2}{\sigma_J}\sum_{n=1}^{N}\frac{I_1(u[n])}{I_0(u[n])}u[n] \quad (14)$$

$$J_{11} = \frac{N}{\sigma_J^2} - \frac{N}{\sigma_J^4} \times \frac{1}{2\Pi\sigma_J^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{11} \times \exp\left(-\frac{x^2[n]+y^2[n]}{2\sigma_j^2}\right) dx[n]dy[n]$$

………. (15)

$$j_{12} = j_{21} = -\frac{2AN}{\sigma_J^3} - \frac{2AN}{\sigma_J^5} \times \frac{1}{2\pi\sigma_J^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{12} \times \exp\left(-\frac{x^2[n]+y^2[n]}{2\sigma_j^2}\right) dx[n]dy[n] \quad (16)$$

$$j_{22} = -\frac{2N}{\sigma_J^2} + \frac{3A^2 N}{\sigma_J^4} + \frac{N}{\sigma_J^4} \times \frac{1}{2\pi\sigma_J^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{22} \times \exp\left(-\frac{x^2[n]+y^2[n]}{2\sigma_j^2}\right) dx[n]dy[n] \quad (17)$$

Where var denotes variance and **J**mn −1 denotes the m, n^{th} element of inverse matrix **J**−1. Thus, inequalities (16,17) provide the performance limits when the boresight and jitter are jointly estimated. They can be numerically evaluated, preferably in the log domain for better numerical behaviour. If only boresight is estimated for a known jitter, we get a bound var_A ˆ 1/$J$11. Similarly, when the boresight is known, the bound on jitter estimation variance is varˆ j1/$J$22. The bounds for joint parameter estimation are higher than individual parameter estimation. In the special case of zero boresight, a closedform expression for CRLB on jitter estimate can be obtained by substituting A=0 into Eq. (13,14). This is given by

$$\mathrm{var}(\hat{\sigma}_j) \geq \sigma_j^2 / 4N \quad (18)$$

WHERE

$$P(x[n], y[n]) = \frac{1}{2\pi\sigma_j^2}\exp\left[\frac{-(x^2[n] + y^{2[n]})}{2\sigma_j^2}\right]$$

$$CRB(\|\gamma\|) = \frac{\sigma_{tp}^2}{2} tr[(V_l^T H^T P \frac{1}{S} H V_l)^{-1}]$$

$$CRB(\|\rho\|) = \frac{\sigma_{\varpi}^2}{2} tr\left[(V_n^t H^T P \frac{1}{S} H V_n)^{-1}\right] / \|\gamma\|^2$$

$$CRB(\|\gamma\|) \big/ \|\gamma\|^2 > CRB(\|\rho\|)$$

$$H^T P \frac{1}{S} H \begin{bmatrix} RE(\rho) \\ IM(\rho) \end{bmatrix} = 0$$

$$J_\eta = \frac{2}{\sigma_{\omega.C}^2} B^T B$$

WHERE

$$B = \begin{bmatrix} \mathrm{Re}(Q) & \mathrm{Im}(Q) & \mathrm{Re}(F) & \mathrm{Im}(F) \\ -\mathrm{Im}(Q) & \mathrm{Re}(Q) & -\mathrm{Im}(F) & \mathrm{Re}(F) \end{bmatrix}$$

$$Q = I_N \otimes \lfloor C_1 h_1, ..., C_P h_p \rfloor$$

$$F = \left[F_1^T, ..., F_N^T\right]^T$$

$$F_N = \lfloor s_1(n) C_1, ..., s_p(n) C_P \rfloor$$

$$CRB(\|\gamma_C\|) = \frac{\sigma_{\omega,C}^2}{2} tr\left[(V_{l,c}^T Q^T P \frac{1}{F} Q V_{l,c})^{-1}\right]$$

$$CRB(\|\rho_C\|) = \frac{\sigma^2_{\omega,C}}{2} tr\left[\left(V^T_{n,c} Q^T P \frac{1}{F} Q V_{n,c}\right)^{-1}\right] / \|\gamma_c\|^2$$

$$\eta^T = \begin{bmatrix} \mathrm{Re}(s_1)^T, ..., \mathrm{Re}(s_N)^T, \mathrm{Im}(s_1)^T, ..., \mathrm{Im}(s_N)^T, \mathrm{Re}(h_1)^T \\ , ..., \mathrm{Re}(h_p)^T, \mathrm{Im}(h_1)^T, ..., \mathrm{Im}(h_p)^T \end{bmatrix}$$

Figure 1 shows the two CRBs and experimental mean square errors for constrained symbol estimation, versus *SNR* where *SNR* = **20 10glo(l/hlluk/u,)** (dB). The mean squared errors of the constrained channel response estimation and the corresponding CRBs versus *SNR* are shown in Figure 2. The mean squared error of the constrained channel response estimation is defined in the same way **as** that of the constrained symbol estimation. It can be seen that the relative statistical efficiency under the two constraints remain the same over a wide range of *SNR*. For symbol estimation, at *SNR* below **30** dB, the performance breaks away from the CRBs. For channel response estimation, it is interesting to note that the performance becomes better than th& CRBs at *SNR* below **20** dB, and the difference can be **as** large **as** around **4** dB at *SNR* **='U** *dB*. Such a phenomenon was reported in **[4]** and recently in [8]. **As** explained in **[11],** this peculiar observation takes place because an upper hound on the variance of a norm-constrained error. goes below CRBs.

$$U_0^H R_\omega^{-1/2} C_n H = 0, \quad n = 1, ..., N \tag{19}$$

where **C***n* is the *J*(*N*+*L*) ×*J*(*L*+1) Toeplitz matrix with the first column **e**(*n*−1)*J*+1 and **e***p* is defined as the *p*th column of **I***J*(*N*+*L*). The first row of **C***n* is [1, **0**1 ×(*J*(*L*+1)−1)] for *n* = 1 and is **0**1 ×*J*(*L*+1) for *n* ≥ 2.

Define

$$K = \left[C_1^H R_\omega^{-1/2} U_0, C_2^H R_\omega^{-1/2} U_0, ..., C_N^H R_\omega^{-1/2} U_0\right] \tag{20}$$

The channel matrix **H** could be estimated from $K^H H = 0$. Therefore, the estimate of **H**, denoted as ˆ**H**, is a basis matrix of the orthogonal complement space of ***K***. We will show later that the dimension of the orthogonal complement space of ***K*** is exactly *K*. Therefore, ˆ**H** can be obtained from left singular vectors of ***K*** and is away from the true **H** by an unknown matrix **B** , mathematically represented by following equations

$$\hat{H} = HB \tag{21}$$

$$G = \hat{G}\left(I_N \otimes B^{-1}\right) \tag{22}$$

$$Z_i = \hat{G}(I_N \otimes B^{-1}) d_i + \eta_i \tag{23}$$

$$\tilde{x}_i(n) = \left[\tilde{x}_i^{(1)}(n), \tilde{x}_i^{(2)}(n), ..., \tilde{x}_i^{(j)}(n)\right]^T \tag{24}$$

$$s_i(n) = \left[s_i^{(1)}(n), s_i^{(2)}(n), ..., s_i^{(k)}(n)\right]^T \tag{25}$$

$$\omega_i(n) = [\omega_i^{(1)}(n), \omega_i^{(2)}(n), ..., \omega_i^{(J)}(n)]^T \tag{26}$$

$$\tilde{H}(n) = \begin{bmatrix} \tilde{h}^{(1,1)}(n) & \cdots & \tilde{h}^{(1,K)}(n) \\ \vdots & \ddots & \vdots \\ \tilde{h}^{(J,1)}(n) & \cdots & \tilde{h}^{(J,K)}(n) \end{bmatrix} \tag{27}$$

$$\tilde{X}_i(n) = \tilde{H}(n) s_i(n) + \omega_i(n) \tag{28}$$

II. RESULTS:

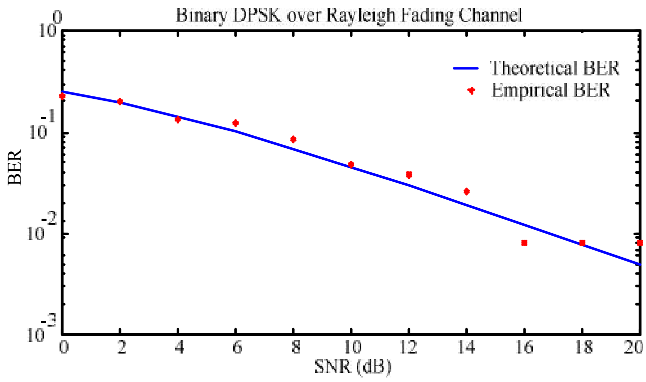


Fig :3 SNR estimation **linear constraints**

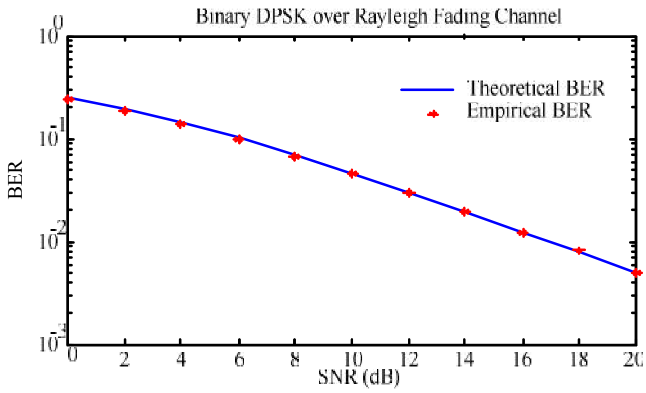


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Fig :4 **norm constraints**

From figure 3, it is clearly observed that Cramer-Rao bounds under linear constraint method the estimated SNR value could not fallow the theoretical results. But figure 4 portrays that Cramer-Rao bounds under Norm constraints method the estimated SNR value fallows the theoretical results. So that the norm constraints method is suitable for SNR estimation for blind channel.

Conclusion

In this paper it shows that for random process (jitter variance) is applied to the **linear and norm constraints the SNR was calculated** and it shows good performance given as graph representation like **norm constraints greater than linear constraints** it is suggested that it can able to apply for SNR estimation to the blind channel

References:

[1]. I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Bell Labs Technical Memorandum*, 1995.

[2]. E. de Carvalho, J. Cioffi, and D. Slock, "Cram´er-rao bounds for blind multichannel estimation," in *Proc. IEEE GLOBECOM'00*, San Francisco, USA,

[3]. F. Li, H. Liu, and R. J. Vaccaro, "Performance analysis for DOA estimation Nov. 2000, vol. 2, pp. 1036-1040.

[4]. Hua; Y., "Fast maximum likelihood for blind identification of multiple FIR channels," *IEEE Dansactions on Signal Processing,* Vol. 44, No. **3,** pp. **661-672,** March **1996.**

[5]. Stoica: P.a,n d Ng, B. C., "Chapter **7:** Performance bounds for blind channel estimation," *Signal Processing Advances in Warelcss Communicutions,* Vol. **2,** P. Stoics: ***G.*** Giaunakis, **Y.** Hua, and L. Tong, Eds. Upper Saddle river, NJ:Prentice Hall, *2000.*

[6]. Stoica, P., and **Ng,** B. C., "On the Cramer-Rao bound under parametric constraints," *IEEE Signal Processing Letters,* Vol. *5,* No. **7,** pp. **177-179, 1998.**

[7]. 'Liu, H., and Xu, *G.,* "A deterministic approach to blind symbol estimation," *IEEE Signal Processing Letters,* Vol. 1, No. 12, pp. 205-207, 1994.

[8]. A. Gorokhov and P. Loubaton, "Subspace-based techniques for blind separation of convolutive mixtures with temporally correlated sources," *IEEE Trans. Circuits Syst. I*, vol. 44, pp. 813-820, Sept. 1997.

[9]. Hua; Y., "Fast maximum likelihood for blind identification of multiple FIR channels," *IEEE Dansactions.*

[10]. Hua, **Y.,** and Wax: M., "Strict identifiability of multiple FIR channels driven by an unknown arbitrary sequence," *IEEE Dansactions on Signal Processing,* Vol. **44,** No. **3,** pp. **756-759, 1996.**

[11] G. Lukesh, S. Chandler and D. G. Voelz, Appl. Opt. **39**, 1359 (2000).

[12]. D. K. Borah, D. Voelz, and S. Basu, Appl. Opt. **45**, 2504 (2006).

[13]. A. Medles and D. Slock, "Linear precoding for spatial multiplexing MIMO systems: blind channel estimation aspects," in *Proc. IEEE ICC'02*, New York, USA, vol. 1, pp. 401-405, 28 April-2 May 2002.

[14]. K. Abed-Meraim, P. Loubaton, and E. Moulines, "A subspace algorithmfor certain blind identification problem," *IEEE Trans. Inform. Theory*,vol. 32, pp. 499-511, Apr. 1997.

[15]. A. Gorokhov and P. Loubaton, "Subspace-based techniques for blind separation of convolutive mixtures with temporally correlated sources," *IEEE Trans. Circuits Syst. I*, vol. 44, pp. 813-820, Sept. 1997.

[16]. E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Trans Inform. Theory*, vol. 45, pp. 1639-1642, July 1999.

[17]. "Radio broadcasing system, digital audio broadcasting (DAB) to mobile, portable, and fixed receiver," Eur. Telecommun. Stand. Inst., Sophia-Antipolis, Valbonne, France, ETS 300 401, 1995-1997.

[18]. "Digital broadcasting system television, sound, and data services; framing structure, channel coding, and modulation digital terrestrial television," Eur. Telecommun. Stand. Inst., Sophia-Antipolis, Valbonne, France, ETS 300 744, 1996.